Multi-kHz mixture fraction imaging in turbulent jets using planar Rayleigh scattering

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Abstract In this study, we describe the development of two-dimensional, high repetition-rate (10-kHz) Rayleigh scattering imaging as applied to turbulent flows. In particular, we report what we believe to be the first sets of high-speed 2D Rayleigh scattering images in turbulent non-reacting jets, yielding temporally correlated image sequences of the instantaneous mixture fraction field. Results are presented for turbulent jets of propane issuing into a low-speed coflow of air at jet-exit Reynolds numbers of 10,000, 15,000, and 30,000 at various axial positions downstream of the jet exit. The quantitative high-speed mixture fraction measurements are facilitated by the use of a calibrated, un-intensified, high-resolution CMOS camera in conjunction with a unique high-energy, high-repetition rate pulse-burst laser system (PBLNS) at Ohio State, which yields output energies of ~200 mJ/pulse at 532 nm with 100-μs laser pulse spacing. The quality, accuracy, and resolution of the imaging system and the resulting image sets are assessed by (1) comparing the mean mixture fraction results to known scaling laws for turbulent jets, (2) comparing instantaneous images/mixture fraction profiles acquired simultaneously with the high-speed CMOS camera and a well-characterized, high-quantum efficiency CCD camera, and (3) comparing statistical quantities such as the probability density function of the mixture fraction results using the high-speed CMOS camera and the CCD camera. Results indicate accurate mixture fraction measurements and a high potential for accurately measuring mixture fraction gradients in both time and space.

1 Introduction

Turbulent mixing is a subject of broad interest in numerous engineering applications and natural processes, where some of the most commonly encountered flows deal with the mixing of a scalar quantity by turbulent motion. Such examples include the mixing of fuel and oxidizer in combustion systems using non-premixed or partially premixed reactant streams, dispersal of pollutants or irritants in the turbulent atmosphere, and the mixing of hot engine exhaust in the cool atmosphere (contrails and stealth applications). In the case of combustion systems, the rate at which the fuel and oxidizer mix governs the rate of chemical reaction. The turbulent mixing field can be uniquely characterized by the mixture fraction, $\xi$, and for laminar flamelet theory (e.g., [1, 2]), the turbulent flame structure can be uniquely related back to $\xi$ and its derivatives. Thus, the mixture fraction can be considered one of the most fundamental and perhaps one of the most important scalars in non-premixed and partially premixed combustion. For non-premixed and partially premixed systems, the rate of molecular mixing is characterized by the scalar dissipation rate, $\chi = 2D(\nabla \xi \cdot \nabla \xi)$, where $D$ is the mass diffusivity and $\xi$ is defined as

$$\xi = \frac{\beta - \beta_{ox}}{\beta_{fuel} - \beta_{ox}}.$$  \hspace{1cm} (1)

In (1) $\beta$ is any conserved scalar, such as elemental mass fraction, and the subscripted-terms represent the “amount” originating from the fuel and oxidizer streams, respectively. Because of the importance of $\xi$ and $\chi$ in turbulent non-premixed and partially premixed combustion, a key focus
of experimental fluid dynamics and combustion research for the past two decades has been measuring these quantities in both non-reacting and reacting flows (e.g., [3–19]). It has been widely regarded that the initial steps in understanding turbulent mixing in combustion environments will stem from an increase in the understanding of the physics of non-reacting turbulent flows. In this regard, substantial attention has been given to developing advanced quantitative laser-based imaging diagnostics, which allow access to detailed topographical and statistical information concerning turbulent mixing. As examples, Prasad and Sreenivasan [4], D’Ahm et al. [5], and Buch and D’Ahm [6] used planar laser-induced fluorescence (PLIF) diagnostics to measure conserved scalar mixing in turbulent liquid-phase flows. For gas-phase turbulent flows, Namazian et al. [3] used planar Raman scattering, while Buch and D’Ahm [7], Everest et al. [8], Feikema et al. [9], Su and Clemens [10, 11], and Frank and Kaiser [12] used highly resolved planar Rayleigh scattering to measure the conserved scalar field and its associated gradients. These measurements have been very important in detailing the turbulent scalar mixing process including elucidating the organizational structure and statistical nature and determining mean scaling laws, mixing layer thicknesses, and length scales of turbulent non-reacting flows.

Although gas-phase mixing measurements (which are of primary interest when considering applications relevant to combustion environments) have been performed in both two- and three-dimensions, they have been limited in temporal resolution, i.e., any two consecutive images have been temporally uncorrelated. Recognizing that turbulence is inherently stochastic, mixing processes must be resolved in both space and time in order to capture the dynamic nature of turbulent flows. This requirement dictates that the data acquisition of two- and three-dimensional fields is at much faster rates than typical time scales of the turbulent processes (\textgreater;1 kHz). In this study, we will describe the use of pulse-burst laser technology (e.g., [20–24]) to produce a series of high-energy laser pulses that can be used for high-speed (10-kHz) planar Rayleigh scattering imaging. Specifically, this work will focus on using Rayleigh scattering as an approach to measuring temporally correlated 2D image sequences of the mixture fraction field in highly turbulent non-reacting jets. Because of the high pulse energies available, a user-calibrated, un-intensified high-speed CMOS camera is used, resulting in very high-quality image sequences of the two-dimensional mixture fraction field. To the authors’ knowledge, these are the first sets of high-speed 2D Rayleigh scattering images in turbulent flows, which resolve the temporally fluctuating nature of turbulent mixing.

A subsequent manuscript will focus on using high-speed Rayleigh scattering imaging to measure temporally correlated 2D image sequences of the temperature field in turbulent non-premixed flames.

2 Previous high-speed imaging

Non-intrusive optical methods have the potential to resolve pertinent physical processes at the smallest length and time scales; however, new experimental tools are required to provide currently unavailable data that are both spatially and temporally resolved. Typical signal levels for gas-phase measurements are sufficiently small such that high-power, pulsed laser sources are required. Commercially available high-energy systems including Q-switched solid state lasers such as Nd:YAG or gas lasers such as excimers are limited to pulse repetition-rates of 10 to 300 Hz, while commercially available high-repetition-rate lasers such as Nd:YLF, Nd:YVO₄, and copper vapor lasers have been limited to low pulse energies. With recent advances in solid-state diode-pumped lasers and CMOS-camera technology, some common laser imaging techniques such as Mie scattering, particle imaging velocimetry (PIV) and PLIF have been demonstrated at multi-kilohertz acquisition rates (e.g., [25–41]). While high-speed PIV measurements have been demonstrated previously (e.g., [25–31]), scalar measurements (e.g., temperature, species concentrations, mixture fraction) remain much more challenging. The low pulse energies available from commercially available high-repetition rate laser systems are prohibitive to 2D Raman and Rayleigh scattering diagnostics, leaving PLIF as the only applicable technique that has been readily used for real-time, multi-dimensional monitoring of flowfield scalars. Recently, Kittler and Dreizler [32] and the group at DLR [31, 33–35] have demonstrated continuous imaging of the OH radical at multi-kHz repetition rates using a frequency-doubled, Nd:YLF-pumped dye laser that is similar to conventional low-repetition-rate systems. In addition, Paa et al. [36] used a frequency-tripled Yb:YAG at 343 nm to excite "hot bands" of the OH radical at 1 kHz and Cundy and Sick [37] used a frequency-quadrupled Nd:YLF at 263 nm to measure OH at 1 kHz.

Relevant to the mixture fraction imaging presented in the current work are the high-speed–scalar imaging studies of Sick and co-workers (e.g., [38–40]) and Gordon et al. [41], who used tracer LIF to measure fuel-air ratios in non-reacting engine environments and mixture fraction in unsteady non-reacting jets, respectively. Tracer LIF denotes the procedure of seeding a fluid with a representative marker that ideally does not affect the fluid properties and emits fluorescence under laser excitation. In this manner, the LIF signal marks the location of the fluid of interest. In the referenced work of Sick and co-workers [38–40], biacetyl (C₄H₆O₂) was chosen as a tracer of iso-octane in cold-firing engines and was excited by the 3rd harmonic output of a diode-pumped Nd:YAG laser at 355 nm operating at 12 kHz. In the work of Gordon et al. [41], acetone (C₃H₆O) was seeded into an air jet issuing into a co-flowing stream of air.
Acetone fluorescence was excited at 266 nm, which was the frequency-quadrupled output of a Nd:YVO₄ diode-pumped slab laser operating at 9.5 kHz. Under these conditions, the local acetone fluorescence signal was related back to the fraction of fluid concentration exiting the nozzle, which was then used to derive the local mixture fraction.

While great care was taken to place the scalar measurements on an absolute scale in both of the previous studies, it should be noted that the reduction of LIF signals into quantitative scalar concentrations is not trivial. PLIF signals are dependent on a number of factors including temperature, electronic (collisional) quenching, and complex photo-physics (spectroscopic considerations). In addition, the tracer seeding level must be carefully controlled and each image must be corrected for laser absorption across the two-dimensional domain. For turbulent flows with spatially and temporally varying concentrations, this may prove difficult. As an example, in the study by Gordon et al. [41], absorption corrections approaching 25% were necessary when using 30% acetone in the jet (this high level of seeding was used to achieve high signal levels). Lower seed concentrations will reduce the absorption correction, but at the expense of collected signal and the signal-to-noise ratio. Finally, it is mentioned that because of the low pulse energies in the UV (~1 mJ) afforded by the commercial laser systems, the LIF signals were collected with an intensified CMOS camera, which is known to decrease the spatial resolution of the measurements and was noted by Gordon et al. [41] in their work.

More recently, Bork et al. [42] demonstrated high-speed 1D Rayleigh scattering measurements in turbulent flames using a commercial, high-repetition-rate 80 W Nd:YAG laser operating at 10 kHz in conjunction with an un-intensified CMOS camera and an optimized light collection system. While the long record lengths and signal-to-noise presented in that work are quite impressive, the lower pulse energies (~5 mJ at 10-kHz) most likely will not facilitate the extension of the 1D measurement to 2D imaging.

3 Experimental methods
3.1 Rayleigh scattering

For low-repetition rate studies, planar Rayleigh scattering is a widely used imaging technique for mixture fraction measurements in non-reacting flows because of the high measurement sensitivity that may be achieved by issuing a fluid with a large Rayleigh scattering cross section into another fluid with a much lower scattering cross section, i.e., propane or Freon into air (e.g., [7–12, 43]). In addition, the determination of mixture fraction (ξ) from the measured Rayleigh scattering signal (I_RAY) is quite straightforward as shown below. Laser Rayleigh scattering is a non-intrusive diagnostic technique that describes the scattering from molecules whose effective diameter is much less than the wavelength of the incident laser light, where the Rayleigh scattering intensity can be written as

$$I_{RAY} = A I_0 n \sigma_{mix}$$  \hspace{1cm} (2)

In (2), A is a constant describing collection volume and efficiency of the optical setup, I_0 is the incident laser intensity, n is the number density, and \(\sigma_{mix}\) is the mixture-averaged differential cross section defined as \(\sigma_{mix} = \sum_{i=1}^{N} X_i \sigma_i\). \(X_i\) and \(\sigma_i\) are the mole fraction and differential cross section of species i, respectively. For a flow that is isothermal and isobaric, changes in the scattered intensity, I_RAY are due to variations in \(\sigma_{mix}\). For a two-stream mixing process (jet fluid issuing into air), (2) can be re-written as

$$I_{RAY} = A I_0 n [X_j \sigma_j + (1 - X_j) \sigma_A]$$  \hspace{1cm} (3)

where \(X_j\) is the mole fraction of the jet fluid at any point in the flow, \(\sigma_j\) is the differential cross section of the jet fluid, and \(\sigma_A\) is the differential cross section of air. Equation (3) can be re-arranged such that the local mole fraction of the jet-fluid (\(X_j\)) is determined from the Rayleigh scattering signal (\(I_{RAY}\)) at any point in the flow and measured calibration signals from pure jet-fluid (\(I_j\)) and pure air (\(I_A\)):

$$X_j = \frac{I_{RAY} - I_A}{I_j - I_A}$$  \hspace{1cm} (4)

Subsequently, the mixture fraction is determined from

$$\xi = \frac{Y_j}{Y_{j,0}} = \frac{X_j W_j}{X_j W_j + (1 - X_j) W_A}$$  \hspace{1cm} (5)

where \(Y_j\) is the mass fraction of the jet fluid at any point in the flow, and \(Y_{j,0}\) is the mass fraction of the jet fluid originating from the source \((Y_{j,0} = 1)\). \(W_j\) is the molecular weight of the jet fluid, and \(W_A\) is the molecular weight of the air.

Although frequently utilized at low-repetition rates, planar Rayleigh scattering requires pulse energies that currently are not available from commercial high-repetition-rate laser systems. In the next section, we describe a custom pulse-burst laser system at Ohio State that produces a series of high-energy laser pulses with micro-second level temporal spacing, allowing high-speed (multi-kilohertz) planar Rayleigh scattering imaging.

3.2 Pulse-burst laser system at OSU

The pulse-burst laser system at Ohio State, shown schematically in Fig. 1a, has been described in detail previously in Ref. [20] and thus will only be described briefly here. The laser system is a master oscillator, power amplifier (MOPA) design, which consists of a single-frequency (<10^3 cm⁻¹)
cw diode-pumped ring laser operating at 1064 nm serving as the primary oscillator, an electro-optic dual pockels cell pulse slicer, and a series of flashlamp-pumped Nd:YAG amplifiers. The cw laser is initially pre-amplified in a double-pass variable pulse width (0.3–2.0 ms) flashlamp-pumped amplifier and then formed into a “burst” of laser pulses by rapidly rotating the polarization of the pre-amplified pulse by one of two pockels cells as described by Wu et al. [44]. In the present experiment, the slicing process creates a train of 10-ns wide pulses which are separated by 100 µs, corresponding to a repetition rate of 10 kHz. We note, however, that this system has been used previously at repetition rates up to 1 MHz (e.g., [20, 22, 45]). The pulse train is then further amplified by a series of four additional flashlamp-pumped Nd:YAG amplifiers, resulting in a system gain of \( \sim 3 \times 10^8 \). The pulse trains are limited in the present study to 1 ms, which corresponds to ten temporally sequential laser pulses and images. However, a new system is under development that will produce a 20 + ms pulse train, corresponding to >200 temporally sequential laser pulses/images at a 10-kHz acquisition rate.

To reduce amplified spontaneous emission (ASE) buildup in the system, a phase conjugate mirror (PCM) is placed between amplification stages 3 and 4 [20]. The PCM is an optical cell filled with a high index-of refraction liquid (FC-75) that uses the principle of stimulated Brillouin scattering (SBS) to act as an intensity filter and break the unwanted ASE growth. In addition, the SBS PCM eliminates the low-intensity pedestal which is superimposed on the desired pulse-burst sequence because of the finite on/off contrast ratio of the pockels cell pulse slicer. When the pump beam intensity is above a minimum threshold, a coherent beam is backscattered at 180° (with a small 350 MHz frequency shift) and the desired high-intensity laser burst is backscattered toward the final amplifier stages, while the sources of low-intensity background (e.g., ASE) do not exceed minimum threshold and pass through the PCM cell to a beam dump. Because the frequency shift of the beam is only 350 MHz, there is no frequency mismatch with the final amplifier stages. Finally, the series of 1064-nm laser pulses are frequency-doubled to 532 nm with a KD\(^+\)P crystal, which is convectively cooled. In the current work, the individual energies of the 532-nm pulses are approximately 200 mJ. An example series of 532-nm pulses are shown in Fig. 1(b), where the intensity difference between the pulses is less than 8%. Previously, this system has been used to perform high-repetition-rate (10 to 50 kHz) OH and CH PLIF imaging in turbulent flames (e.g., [21, 24]), NO PLIF imaging in hypersonic flows [45], and Raman scattering measurements in isothermal turbulent jets [23].

3.3 Optical arrangement

A schematic of the optical arrangement used for these experiments is given in Fig. 2. In the high-speed imaging arrangement, the 532-nm output of the pulse-burst laser system
(PBLS) is initially passed through a thin film polarizer (TFP) to ensure that only vertically polarized laser light transmits to the test-section because of the polarization-dependence of Rayleigh scattering. The vertically polarized light (approximately 95% of the initial pulse energy) is formed into a 40 × 0.16 mm² laser sheet via a combination of a single plano-concave cylindrical lens ($f = -100$ mm) and a plano-concave spherical lens ($f = 750$ mm). The laser-sheet thickness (reported as the $1/e^2$ value) was determined by first removing the cylindrical lens and measuring the beam diameter at the focal point of the probe volume using Rayleigh scattering imaging from ambient air.

The multi-kHz Rayleigh scattering images are collected with a single high-speed CMOS camera (Vision Research, Phantom v710) outfitted with an 85 mm $f/1.2$ Nikon camera lens operating at 10,000 frames per second with an effective resolution of 1024 × 664 pixels. The resulting magnification is approximately 1:3 with no barrel distortion, vignetting, or spherical aberration apparent. This configuration yields an imaged field-of-view of approximately 60 × 40 mm² with an in-plane spatial resolution of 60 μm. Thus, the out-of-plane spatial resolution, as determined by the laser-sheet thickness, determines the limiting spatial resolution of the measurements. This will be compared with the smallest physical length scales of interest in Sect. 3.6 to examine whether the smallest flow features can be adequately resolved. The synchronization of the camera acquisition to the PBLS output is controlled using a commercially available high-speed camera controller (HSC, LaVision, Inc.). As described below in Sect. 3.5, the laser pulse energy referencing is performed using Rayleigh scattering from a uniform region of the co-flowing air, thus an additional detector to monitor the energy fluctuations is not required.

In a second set of experiments, the mixture fraction results from the CMOS-based imaging system are compared directly to that of a CCD-based imaging system to demonstrate the ability to generate high-quality, quantitative mixture fraction results using a user-calibrated CMOS camera (the calibration of the CMOS camera is discussed below). For this configuration, the output of a single, commercially available, Q-switched, Nd:YAG laser (Spectra-Physics, Pro 290-10) operating at 10 Hz is frequency-doubled to 532 nm and directed through the same optical arrangement as noted above for the PBLS. The Rayleigh scattering signal is simultaneously detected by both the CMOS camera and a high-quality, high quantum efficiency, scientific-grade CCD camera (PCO Sensicam) as shown in Fig. 2. The CMOS camera and lens combination remains the same as that described for the high-speed applications, while the CCD camera also is outfitted with an 85 mm $f/1.2$ Nikon camera lens. The CCD camera is arranged such that the field-of-view is identical to that of the CMOS camera, resulting in a magnification of approximately 1:9 owing to the smaller size of the CCD sensor (8.9 × 6.7 mm²). Image acquisition occurs at a rate of 5 frames per second and is synchronized by the HSC.

### 3.4 Turbulent jet conditions

In this study, we consider the turbulent flow of a propane jet issuing from an 8-mm-diameter circular tube into a 300 mm × 300 mm co-flowing stream of air. Three cases are considered where the propane exits the tube at 6, 9, and 18 m/s into a 0.7 m/s co-flow, corresponding to Reynolds numbers of 10,000, 15,000, and 30,000 based on tube diameter. Propane is chosen for its large Rayleigh scattering cross section (approximately 13 times that of air), which results in a high signal-to-noise ratio for the acquired images. The co-flowing air is supplied by a forced-air blower (rated at 14 m³/min), which passes through a series of perforated plates (1-mm hole diameter, 45% open area), a HEPA filter, and a 3-mm hexagonal cell-size honeycomb flow straightening section for flow conditioning and particulate removal, resulting in a particle-free, laminar, uniform co-flow. Particulate filtering (e.g., dust) is absolutely essential because laser Mie scattering from particles can completely mask the Rayleigh scattering signal.

### 3.5 Image processing and data reduction

Because it is desired to deduce quantitative mixture fraction $\xi(x,t)$ fields, proper reduction of the acquired Rayleigh scattering signals is critical. The data reduction process follows a systematic procedure which encompasses: (1) subtraction of background signals (e.g., darkfield image and stray light scattering subtraction), (2) linearization of the camera, (3) correction for sensor non-uniformity, (4) correction for shot-to-shot energy fluctuations, and (5) correction for the non-uniformity of the laser-sheet intensity distributions. For CMOS sensor-based cameras steps (2) and (3) are particularly important. For the simultaneous CMOS and CCD camera experiments, two additional steps are required: (6) mapping the two image planes onto the same imaging area and (7) matching the Rayleigh scattering signal levels between cameras.

In contrast to well-characterized CCD sensors which have uniform pixel response and an exceptionally high degree of sensor linearity, CMOS sensors have the potential to have an independent, non-linear response for each pixel [46] because each pixel in a CMOS sensor acts as an independent active circuit. Recent papers [47, 48] focusing on the linearity and uniformity of CMOS cameras have appeared in the literature demonstrating the need to carefully calibrate CMOS cameras for quantitative measurements. In this study, we follow the recent suggestions of Weber et al. [48] by performing a pixel-by-pixel correction for CMOS sensor response.
The linearity and non-uniformity of the CMOS camera was tested using an experimental setup similar to that described by Hain et al. [47], which is a calibrated, uniform light source illuminated the CMOS sensor as depicted in Fig. 3a. The illumination source was an Ulbricht sphere (LMT Lichtmeßtechnik; Berlin, Germany) with an aperture \((2r)\) of 70 mm. The Ulbricht sphere provides a well-defined light source with non-uniformities of less than 0.1% over the entire aperture of the diffusing screen. Since the incident illuminance \(E\) (lux = lumens/m²) to the CMOS sensor is calculated as \([47]\)

\[
E = \frac{\pi r^2}{r^2 + d^2} L
\]

where \(d\) is the distance between the sensor and the Ulbricht sphere as shown in Fig. 3a and \(L\) is the luminance (candelas/m²) of the Ulbricht sphere (at the diffusing screen), the luminance level at the sensor can be varied by adjusting \(d\), that is, the distance between the CMOS camera and the Ulbricht sphere. This was accomplished by mounting the CMOS camera on a precision optical rail and systematically increasing (or decreasing) \(d\) in known increments with an accuracy of 0.5 mm. For these experiments, the CMOS sensor is illuminated directly by the Ulbricht sphere without a camera lens and without any additional ambient light (i.e., a dark room).

The linearity of the CMOS sensor is shown in Fig. 3b, where the results represent the average collected signal of 250 images after removing the average background (dark image) signal. Results are shown for four separate pixels (as depicted in the upper left of Fig. 3b), although we note that the results for all 1024 x 664 pixels are available for the image processing. Figure 3b shows that the CMOS sensor is quite linear over a wide range of incident illuminance, which is consistent with previous results [47, 48]. The linear range covers approximately two orders of magnitude, where only the lower ∼50 (out of ∼4030) counts display noticeable non-linearity. Figure 3b also illustrates that there is very little variation between individual pixels in terms of linearity behavior for the four pixels shown, which is also representative of the entire 6.8 x 10⁵ pixel array. Also shown in Fig. 3b are the results from a well-characterized, scientific-grade CCD (PCO Sensicam) as a reference. Over the same range of sensor signal levels (2–98% of max), there are no significant differences in the linearity characteristics of the CMOS and CCD sensors.

The average images obtained using the Ulbricht sphere also are useful for correcting for sensor spatial non-uniformity (i.e., the "whitefield" response function) for a given signal level. Figure 3c shows a line plot which represents a horizontal “slice” at a vertical location corresponding to pixel 332 of an average of 250 images (signal level ~2500 counts). It is observed that the image is highly uniform, with the most extreme non-uniformities (∼3% from average) arising at the edges of the sensor. The degree of (non-)uniformity of each average image was found to be in-

![Fig. 3](image_url)

(a) Schematic of the test setup for determining the linearity and uniformity of the high-speed CMOS camera. (b) Linearity results depicting the dependence on the acquired signal (displayed in terms of signal counts) on \(E \cdot \Delta t\). Results from the CMOS camera are given by the solid data points for four camera pixels (P1–P4) that correspond to the position depicted in the upper left corner of the figure. (c) Line plot at pixel 332 in the vertical direction from a 250-shot average CMOS camera image depicting the small degree of image non-uniformity.
dependent of the incident illuminance, which is consistent
with the small pixel-to-pixel variance in signal linearity as a
function of incident illuminance. Hence, for the results pre-
sented in this paper, the sensor spatial non-uniformity is cor-
rected by a single 250-shot average image obtained at the
product of the incident illuminance and the camera shutter
time \( (E \cdot \Delta t) \) of 0.01.

Shot-to-shot laser pulse energy fluctuations and laser-
sheet intensity non-uniformities were determined directly
from the individual Rayleigh scattering images by measur-
ing the instantaneous intensity magnitude and spatial distrib-
utions in a portion of the image containing only the co-
flowing air stream. For a uniform illumination source with
constant pulse energy, the regions of the co-flowing air (i.e.,
before the laser sheet propagates into a mixture of jet-fluid
and air) should exhibit a constant signal, both in the laser-
sheet spanwise direction encompassing the imaging area and
from one image to the next. Thus, any spanwise or pulse-to-
pulse variation in signal intensity (in the region of pure air)
directly corresponds to a local variation in laser intensity and
can be easily accounted for.

For the simultaneous CMOS/CCD imaging experiments,
the image pairs must be mapped onto the same imaging
area. Mapping the image pairs to the same imaging area
and spatial resolution requires translation, rotation, and sub-
sampling of the higher-resolution images. This is accom-
plished by first imaging a thin, transparent calibration target
placed within the measurement plane with both the CCD and
CMOS cameras. The ‘mapping’ of the CCD image onto the
spatial domain of the CMOS camera is then executed within
the commercially available DaVis software from LaVision,
Inc. Next, the signal levels between any CMOS/CCD image
pair are matched by ensuring that the signals occurring from
a uniform region of co-flowing air are equivalent. Because
the Rayleigh scattering captured by both cameras originates
from a single laser sheet, the degree of spatial overlap of
the two image pairs can be assessed by examining the laser-
sheet intensity distribution within the co-flowing air region.
For these measurements, no differences in the laser-sheet in-
tensity distributions are found as a function of spanwise di-
rection when comparing the CCD and CMOS images. Thus
it is concluded that the two cameras image the same spa-
tial region to within less than one pixel (60 \( \mu \text{m} \)). Finally,
we note that the images presented in this study have had no
additional pixel binning or filtering applied, thus the final
image size for the Rayleigh images are 1024 \( \times \) 664 pixels
with an in-plane grid spacing (spatial resolution) of 60 \( \mu \text{m} \).

3.6 Some comments on spatial and temporal resolution

In order to accurately measure scalar properties including
fluctuations, gradients, and dissipation rates, the combina-
tion of the measurement probe (i.e. the pulsed laser) and the
experimental setup must produce sufficient spatial and temp-
oral resolution such that the smallest characteristic length
and time scales are resolved. It is generally accepted that this
requires that the probe volume and camera system resolution
are smaller than the smallest scales at which scalar fluctua-
tions occur and the sampling rate is faster than the highest
scalar fluctuation frequency present in the flowfield.

The finest length scale of a scalar field was defined by
Batchelor [49] on the basis of dimensional reasoning as

\[
\lambda_B = \left( \frac{vD}{\langle \epsilon \rangle} \right)^{1/4} = \lambda_K \Sigma_c^{-1/2}
\]

(7)

where \( v \) is the kinematic viscosity, \( D \) is the mass diffusivity,
\( \langle \epsilon \rangle \) is the mean rate of kinetic energy dissipation, \( \lambda_K \) is the
Kolmogorov microscale, and \( \Sigma_c = v/D \) is the Schmidt num-
ber. The Batchelor scale can be cast in terms of outer-scale
flow variables by considering the relation \( \langle \epsilon \rangle = C U^3/\delta \)
from Taylor [50], in combination with the measurements of
\( \langle \epsilon \rangle \) in turbulent non-reacting jets by Antonia et al. [51],
where \( C \) is a constant, \( U \) is a characteristic outer-scale ve-
locity and \( \delta \) is a characteristic length scale, which for a jet
is taken as the full width at half maximum of the velocity pro-
file. If the characteristic velocity \( U \) is taken as the centerline
velocity \((U_c)\), the Batchelor scale can be expressed as

\[
\lambda_B = 2.36 \delta \text{Re}_\delta^{-3/4} \Sigma_c^{-1/2}
\]

(8)

where \( \text{Re}_\delta = U_c \delta /v \).

Since \( \lambda_B \) was defined based on dimensional reason-
ning only, several studies have introduced an alternative
length scale, \( \lambda_D = A \delta \text{Re}_\delta^{-3/4} \Sigma_c^{-1/2} \), which is the dissi-
pative length scale, which is defined as the ratio of smallest
length scale at which scalar mixing occurs to the largest fluid
mechanical length scale [6, 7, 10, 11]. Based on measured
scalar dissipation rate layer thicknesses, Buch and Dähn [6, 
7] have reported the scaling constant \( A \) as 11.2. It is noted
that the dissipative length scale is proportional to the Batch-
elor length scale, but somewhat larger.

For highly turbulent flows, the highest scalar fluctuation
frequency would presumably correspond to the convective
Batchelor frequency, \( f_B = \langle U \rangle / (2\pi \lambda_B) \), where \( \langle U \rangle \) is the
local mean velocity. In addition, it may be reasonable to
define a “dissipative” frequency, \( f_D = \langle U \rangle / (2\pi \lambda_D) \), which
may be more representative of the rate at which scalar mix-
ing occurs. According to Nyquist–Shannon sampling theory,
for high-repetition rate studies, the measurements should be
acquired at sampling rates that are at least twice as fast as
the appropriate characteristic frequency.

Recently, spatial resolution requirements were tested by
Mi and Nathan [52] in non-reacting, heated jets. Their re-
results showed that scalar measurements and their variance
were adequately resolved if the spatial resolution of the ex-
perimental system was less than \( \lambda_D \), but the requirement for
scalar dissipation rate measurements was to resolve one to two times the Batchelor scale. A similar argument could be proposed for temporal resolution, that is, depending on the quantity of interest (i.e., scalar field, scalar variance, or scalar dissipation rate) to “track” in time, the necessary temporal resolution would be expected to fall between $f_B$ and $f_D$. Figure 4(a) shows an estimate of both $\lambda_B$ and $\lambda_D$ at the centerline of the turbulent jet as a function of the normalized axial position, $x/d$, where $x$ is the axial position and $d$ is the tube diameter, for all three Reynolds number cases considered in this study. Similarly, Fig. 4(b) shows an estimation of the range of characteristic frequencies (bounded by $f_B$ and $f_D$) expected along the centerline of the turbulent jet as a function of $x/d$ for all three Reynolds number cases.

As mentioned previously, the camera pixel resolution was 60 $\mu$m based on the combination of the pixel size and system magnification. To test the in-plane spatial resolution, a 100 $\mu$m diameter pinhole was illuminated by a white light source and imaged with the camera. The pinhole was well resolved indicating that the in-plane resolution of the camera was less than 100 $\mu$m and the system resolution would be determined by the out-of-plane spatial resolution, which is determined by the laser-sheet thickness. The laser-sheet thickness (LST) was measured to be 90 $\mu$m based on the FWHM value and 160 $\mu$m based on the $1/e^2$ value. Both values of LST are plotted in Fig. 4(a) in comparison to the smallest physical length scales. We note that for all Reynolds number conditions, $\lambda_D$ is resolved by the measurements, but $\lambda_B$ is mostly likely only resolved at axial locations 20 to 40 times the exit diameter depending on Reynolds number.

The temporal resolution of the measurements is determined by the inter-pulse spacing, which can be varied using the PBLs as described in Sect. 3.2. This is one of the key features of the PBLs, that is, the effective repetition rate of the measurements can be systematically increased as test conditions warrant finer temporal resolution (i.e., measurements near the nozzle exit or with increasing Reynolds number). In this study, all measurements are presented at a 10-kHz acquisition rate (100 $\mu$s spacing) for demonstration purposes.

4 Results and discussion

This section presents sample results obtained using the high-energy laser output from the PBLs to derive temporally correlated image sequences of the mixture fraction ($\xi$) field in turbulent non-reacting jets using planar Rayleigh scattering. These measurements are useful for visualizing the dynamics of scalar mixing in “real time” in turbulent flows. Results are presented from measurements at axial locations corresponding to $x/d = 8$, 12, and 25. The accuracy of the measurements is discussed in detail and is assessed systematically through statistical analysis and comparison with results derived from low-repetition rate measurements (10 Hz) using a well-characterized, scientific-grade CCD camera.

4.1 Example 10-kHz image sequences

Figure 5 shows an example of a full ten-frame temporal sequence of 2D mixture fraction images corresponding to the Re = 10,000 case at an axial location of $x/d = 8$. Each image is centered at $r/d = 0$ and represents a $23 \times 35$ mm field-of-view. Each image has been place on a quantitative scale
(ξ = 0 to 1) using (4) and (5) and the variation in mixture fraction is represented with a gray-scale color map to highlight the high scalar gradients present throughout the jet. The high signal-to-noise ratio (SNR) and resolved small-scale features are evident in the image sequences. To quantify image quality, uniform regions within the jet core (ξ = 1) and in the co-flowing air stream (ξ = 0) provide areas to assess the “single-shot” SNR of the measurements over the full range of measured signals. While regions of co-flowing air appear in every image set, regions of the true jet core (ξ = 1) do not appear in every image set. However, having even one data set allows a proper identification of SNR since it is the “single-shot” SNR that is of interest.

To determine the SNR in these two regions, a 10 × 30 pixel window was chosen within the ξ = 1 regions, and a 10 × 100 pixel region was chosen within the co-flowing air stream (ξ = 0). For the current image sequences, the single-shot SNR for an instantaneous image was found to be approximately 40 in the jet core (ξ = 1) and 9 in the co-flowing air stream (ξ = 0). It is worth noting again that no filtering has been applied to the mixture fraction images. Thus, it is concluded that if a higher SNR is desired, simple 3 × 3 median filtering would increase the SNR of the measurements to ~120 and 27 in the ξ = 1 and ξ = 0 regions, respectively, without degrading the in-plane spatial resolution beyond the limiting spatial resolution of the system, which is the out-of-plane spatial resolution as determined by the laser-sheet thickness. The high SNR over the entire mixture fraction range is a benefit of the Rayleigh scattering method as compared to other tracer-LIF approaches (e.g., [38–41]) that seed a tracer into the “fuel” stream. Simply put, when using the planar Rayleigh method, a signal is obtained from pure air (ξ = 0), unlike tracer-LIF approaches where the signal (and SNR) drop to zero as the mixture fraction approaches zero. This is an important consideration as many practical fuel-air systems (in the context of combustion systems) have a stoichiometric mixture fraction <0.1.

The significance of these images, however, does not lie in the spatial quality (as conventional, low-repetition-rate, single-shot measurements using CCD cameras achieve higher signal-to-noise ratios) but in the temporal correlation. The utility of the time-resolved image sequence is evidenced in the ability to track turbulent flow dynamics, and in particular, scalar mixing, in real time. As an example, the evolution of the mixture fraction topology and small-scale scalar structures are clearly seen, thereby displaying the time-varying nature of turbulent mixing and the steep concentration gradients that turbulent mixing introduces.

Figures 6, 7, 8 show example temporal image sequences for the Re = 10000, 15000, and 30000 flow conditions, re-
Fig. 7 Five-frame, 5-kHz sequence of the mixture fraction field in a turbulent \((Re = 15000)\) non-reacting propane jet issuing into air. Image sequences are shown for three downstream axial positions corresponding to \(x/d = 8, 12, 25\). The images were acquired at 10 kHz, but only every other image (200 \(\mu\)s spacing) is shown for clarity. The field-of-view of the images is 27 mm \(\times\) 42 mm.

Fig. 8 Four-frame, 10-kHz sequence of the mixture fraction field in a turbulent \((Re = 30000)\) non-reacting propane jet issuing into air. Image sequences are shown for three downstream axial positions corresponding to \(x/d = 8, 12, 25\). The field-of-view of the images is 27 mm \(\times\) 42 mm.

respectively, for three different axial locations, all centered at \(r/d = 0\). Each image corresponds to a field-of-view of 27 \(\times\) 42 mm. For the \(Re = 10000\) and \(Re = 15000\) conditions (Figs. 6 and 7), any two successive images are shown with 200 \(\mu\)s spacing. For the \(Re = 30000\) case (Fig. 8), the displayed images are separated by 100 \(\mu\)s because of the fast advection of the scalar structures at this high Reynolds number condition. As expected, for each Reynolds number condition, the scalar field displays finer structures with decreasing axial position and for a given axial position, the structures become finer for increasing Reynolds number. These results give a qualitative indication that the mixture fraction field is adequately spatially resolved, especially for the downstream axial positions and the lower Reynolds number cases. In terms of temporal resolution, the \(Re = 10000\) and 15000 cases display smooth advection of the scalar structure as a function of time for all three axial positions. This indicates that for temporally resolving the mixture fraction field itself (not gradients), sampling rate requirements may not be as stringent as resolving \(f_B\). We note, though, that this is based solely on observation and a true assessment of temporal resolution requirements can be deduced by determining a scalar energy spectrum and determining what portion of the scalar fluctuation energy is captured for a given temporal resolution.

4.2 Comparison with turbulent scaling laws

A preliminary assessment of the accuracy of the mixture fraction measurements is made by comparing both the mean value of the centerline mixture fraction, \(\xi_c\), and the jet width for all three Reynolds number cases and axial positions to known scaling laws for turbulent jets. The mean values are determined from 20 image sets or 200 individual images per Reynolds number per axial distance. As discussed by Tacina and Dahn [53], the centerline value of the mixture fraction is expected to scale as

\[
\xi_c(x) = 5.4(x/d^*)^{-1}
\]

where \(x\) is the axial distance downstream of the exit nozzle and \(d^*\) is the “momentum diameter” which accounts for differences in density between the jet fluid and the ambient air, i.e., \(d^* = (\rho_{jet}/\rho_{air})^{1/2}d\), where \(d\) is the nozzle diameter. In addition, the jet width (e.g., the “outer length scale”)
mixture fraction field in turbulent jets using conventional high-pulse energy lasers (e.g., Nd:YAG) and scientific-grade CCD sensor-based cameras (e.g., 10–12). While both CMOS- and CCD-based detectors convert light into electric charge and into signals, the conversion process and the architecture of the two types of sensors are quite different. Modern CCD cameras exhibit uniform responses, low noise, and high linearity with respect to incident photon-to-electronic signal conversion owing to decades of technological advances driven by the needs of various scientific communities. Thus, measurements using a scientific-grade CCD camera can be used as a “standard” from which to compare to the results obtained using the high-speed CMOS camera.

Briefly, each pixel in a CMOS sensor has an individual charge-to-voltage converter in addition to individual amplifiers and analog-to-digital converters. Consequently, each pixel has less area available for light capture (lower fill factor), each pixel may exhibit a different response for a given number of incident photons, this response may be nonlinear, and the total sensor may exhibit lower uniformity. Each of these facets was discussed in detail in Sect. 3.5 and the high-speed CMOS camera used in the present experiments was characterized in terms of these performance metrics as shown in Fig. 3. Simply stated, CMOS cameras are considered to exhibit higher levels of noise, lower levels of dynamic range, increasing levels of non-uniformity, and increasing levels of sensor non-linearity as compared to their CCD counterparts, with the one advantage of a significantly increased rate of image capture (~1000× faster). However, an underlying question remains: can quantitative scalar measurements be made using a CMOS-based camera?

In this section, mixture fraction measurements obtained simultaneously using a scientific-grade CCD and a high-speed CMOS camera (in conjunction with a single laser source operating at 10 Hz) are examined. The experimental configuration for these measurements was described in Sect. 3.3. The degree in which the CMOS-derived mixture fraction results match those of the CCD-derived results will be used as an indication of the quality of measurements using the present high-speed CMOS-based detection scheme. Figure 10 displays four random pairs (CMOS and CCD results) of the instantaneous mixture fraction fluctuation, \( \xi' = \xi - \langle \xi \rangle \), where \( \xi \) is the instantaneous mixture fraction and \( \langle \xi \rangle \) is the mean mixture fraction obtained from 600 individual images. The mixture fraction fluctuation, \( \xi' \), is displayed to visually highlight the large gradients present within the flowfield and to more easily discern any differences between the results using the two detector types. The image pairs shown correspond to (1) \( \text{Re} = 10000, x/d = 8 \); (2) \( \text{Re} = 10000, x/d = 12 \); (3) \( \text{Re} = 15000, x/d = 8 \); (4) \( \text{Re} = 15000, x/d = 12 \).
Fig. 10 Comparison between instantaneous mixture fraction fluctuation ($\xi'$) images acquired with the high-speed CMOS camera and the scientific-grade CCD camera. Acquisition rate was 10 Hz. The image pairs shown correspond to (1) Re = 10000, $x/d = 8$; (2) Re = 10000, $x/d = 12$; (3) Re = 15000, $x/d = 8$; (4) Re = 15000, $x/d = 12$. The right-most column displays instantaneous radial $\xi'$ profiles across the image pairs located to the left corresponding to the vertical position indicated by the arrow.

Upon visual inspection, there is no discernable difference between the 2D $\xi'$ fields obtained using the CMOS camera and the results using the low-noise, scientific-grade CCD camera. To further test the degree of accuracy of the $\xi'$ results obtained using the CMOS camera, instantaneous $\xi'$ profiles (corresponding to the location denoted by the arrow) from both the CMOS and CCD cameras are compared as shown in the right column of Fig. 10. The two $\xi'$ profiles for each image pair shown display a high-level of agreement and demonstrates the ability to accurately capture large scalar gradients using the CMOS camera. Although, the results obtained using the CMOS camera show a higher level of noise (as expected) and small differences at a few discrete radial locations than the results obtained using the CCD camera, the level of agreement is quite remarkable. In fact, the correlation coefficient between the $\xi'$ profiles derived from the CMOS and CCD cameras is 0.99 for all four images pairs shown in Fig. 10. We also note that some level
of disagreement between the two radial profiles is expected because of the mapping algorithm used to force the two image planes onto the same imaging area.

To statistically assess the CMOS/CCD comparison, the probability density function (PDF) of the centerline mixture fraction ($\xi_c$) is determined for each set of results using the CMOS and CCD cameras as shown in Fig. 11. The PDF of the centerline mixture fraction is determined using a $10 \times 10$ pixel region at the center of the 2D mixture fraction images resulting in a PDF calculated using $6 \times 10^6$ data points. The PDFs for Figs. 11(a)–(d) correspond to the flow conditions (Reynolds number and axial position) of image pairs 1–4, respectively, shown in Fig. 10. The PDFs are displayed over the full range of mixture fraction values ($0 < \xi < 1$) with a bin spacing, $\Delta \xi$, corresponding to 0.02. As shown in Fig. 11, the results of the calculated PDFs for the instantaneous mixture fraction results obtained using the CMOS and CCD cameras show excellent agreement for more than two decades of probability values. From Figs. 10 and 11, it is obvious that high-quality $\xi$ (or $\xi'$) measurements can be obtained using the aforementioned user-calibrated CMOS camera, thus permitting the opportunity for quantitative, multi-kilohertz measurements of the mixture fraction field in turbulent non-reacting jets.

4.4 Scalar gradients and scalar dissipation rate

The previous sections have demonstrated the ability to measure highly resolved, quantitative 2D mixture fraction measurements at multi-kilohertz acquisition rates in turbulent jets using planar Rayleigh scattering. One of the ultimate goals of developing a diagnostic approach for acquiring temporally correlated images of the 2D mixture fraction field is to be able to examine the temporal evolution of the scalar gradient and scalar dissipation rate ($\chi$) fields. Deducing spatially and temporally resolved fields of $\nabla \xi$ and $\chi \sim (\nabla \xi)^2$ is a stringent test of the quality of the acquired Rayleigh scattering images because high levels of noise will result in artificial mixture fraction gradients and once squared (as required for deducing $\chi$) will yield inaccurate dissipation rate results. As a first test at the ability to deduce $\nabla \xi$ and $\chi$, Figs. 12 and 13 display a five-image sequence of the mixture fraction field and the corresponding $|\nabla \xi|^2$ field, which is proportional to the scalar dissipation rate, for the $Re = 10000$ cases at $x/d = 8$ and 12, respectively. Displaying $\ln |\nabla \xi|^2$ reveals the structure at both large and small values of the dissipation rate. It is evident that the majority of the scalar dissipation is organized into thin, “sheet-like” layers, consistent with low-repetition rate results from Buch
Fig. 12 (Upper) Five-frame, 5-kHz sequence of the mixture fraction field in a turbulent (Re = 10000) non-reacting propane jet issuing into air. (Lower) Five-frame, 5-kHz sequence of the scalar dissipation rate field, (Vξ)² calculated from the mixture fraction image sequence shown in the upper row. Images are centered at x/d = 8. The images were acquired at 10 kHz, but only every other image (200 µs spacing) is shown for clarity. The field-of-view of the images is 23 mm × 35 mm

Fig. 13 (Upper) Five-frame, 5-kHz sequence of the mixture fraction field in a turbulent (Re = 10000) non-reacting propane jet issuing into air. (Lower) Five-frame, 5-kHz sequence of the scalar dissipation rate field, (Vξ)² calculated from the mixture fraction image sequence shown in the upper row. Images are centered at x/d = 12. The images were acquired at 10 kHz, but only every other image (200 µs spacing) is shown for clarity. The field-of-view of the images is 23 mm × 35 mm

and Dahm [6, 7], Su and Clemens [10, 11], and Kaiser and Frank [12] using CCD-based detectors. Consistent with previous results at similar downstream regions, the thin dissipation layers maintain their orientation over distances much greater than their width and the strongest layers appear to be oriented at 45° to the jet-axis. No noticeable level of dissipation rate is observed in the co-flowing air region, which should be uniform (i.e., no gradients), but it consists of the lowest measured signal and thus the highest levels of noise. Although this is not an indication of accurate dissipation rate measurements, it does give confidence that the Vξ and (Vξ)² measurements are not dominated by noise. Future work will focus on determining the accuracy of the multi-kilohertz mixture fraction gradient and scalar dissipation rate measurements by comparing simultaneous CCD and CMOS measurements and acquiring scalar dissipation rate measurements in laminar flows with known gradients.

5 Conclusions

In this study, we have described the development of quantitative, two-dimensional, high-repetition-rate (e.g., 10-kHz) Rayleigh scattering imaging as applied to turbulent non-reacting jets to deduce temporally correlated image sequences of the mixture fraction field. High pulse energies (~200 mJ/pulse at 532 nm) are produced from the custom pulse-burst laser system available at Ohio State enabling high-speed (10-kHz acquisition rate) 2D Rayleigh scattering imaging for the first time. Quantitative high-speed imaging is enabled by using a user-calibrated, un-intensified CMOS camera. Simultaneous imaging results using the high-speed CMOS camera and a low-noise, scientific-grade CCD camera show an exceptional level of agreement both in terms of instantaneous mixture fraction profiles (i.e., accurately capturing the mixture fraction gradients) and the comparison of the probability density functions of the centerline mixture fraction for Reynolds numbers cases of 10,000 and
15,000. Both of these results indicate that if carefully calibrated, the associated non-linearity and non-uniformity that characterize CMOS-based cameras can be accounted for and quantitative scalar measurements can be made. Preliminary image sequences of the scalar dissipation rate are also presented demonstrating the potential for accurately measuring the scalar dissipation rate in both space and time.

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