Low-temperature $M=3$ flow deceleration by Lorentz force

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This paper presents results of cold magnetohydrodynamic (MHD) flow deceleration experiments using repetitively pulsed, short pulse duration, high voltage discharge to produce ionization in $M=3$ nitrogen and air flows in the presence of transverse direct current electric field and transverse magnetic field. MHD effect on the flow is detected from the flow static pressure measurements. Retarding Lorentz force applied to the flow produces a static pressure increase of up to 17%–20%, while accelerating force of the same magnitude results in static pressure increase of up to 5%–7%. The measured static pressure changes are compared with modeling calculations using quasi-one-dimensional MHD flow equations. Comparison of the experimental results with the modeling calculations shows that the retarding Lorentz force increases the static pressure rise produced by Joule heating of the flow, while the accelerating Lorentz force reduces the pressure rise. The effect is produced for two possible combinations of the magnetic field and transverse current directions producing the same Lorentz force direction (both for accelerating and retarding force). This demonstrates that the observed static pressure change is indeed due to the MHD interaction, and not due to Joule heating of the flow in the crossed discharge. No discharge polarity effect on the static pressure was detected in the absence of the magnetic field. The fraction of the discharge input power going into Joule heat in nitrogen and dry air, inferred from the present experiments, is low, $\alpha=0.1$, primarily because energy remains frozen in the vibrational energy mode of nitrogen. This result provides first direct evidence of cold supersonic flow deceleration by Lorentz force. © 2006 American Institute of Physics. [DOI: 10.1063/1.2265011]

I. INTRODUCTION

The use of nonequilibrium (low-temperature) magnetohydrodynamics for supersonic flow control and power generation continues to attract considerable interest. Over the last few years, numerous theoretical system studies and modeling calculations in this field have been complemented by experimental results. In particular, experiments at the Ohio State University showed that retarding Lorentz force results in significant density fluctuation increase in a supersonic boundary layer in low-temperature $M=3$ nitrogen and air flows.\(^1,2\) The repetitively pulsed discharge ionization technique has also been used at Princeton University to demonstrate feasibility of magnetohydrodynamic (MHD) power extraction from a cold $M=3$ air flow.\(^2\) Finally, nonequilibrium MHD flow experiments at Wright-Patterson Air Force Base showed that a near-surface glow discharge combined with the magnetic field can be used to control surface pressure on a model in an $M=5$ air flow.\(^4\)

Considerable effort has been made to demonstrate cold supersonic flow acceleration or deceleration by Lorentz force, with the main application being MHD flow control in hypersonic inlets.\(^5\) Recent results obtained at Princeton demonstrate Lorentz force acceleration of a constricted discharge filament sustained near the test section wall in a cold $M=3$ air flow, up to velocities of 1.9 km/s.\(^6\) If the momentum of the accelerated filament is coupled to the flow due to collisions between the charged species and the neutral species, this would result in the boundary layer flow acceleration (the “snowplow” effect). Indeed, 3D compressible Navier-Stokes MHD modeling calculations\(^7\) suggest that Lorentz force flow acceleration and deceleration may be detected at the flow conductivities realized at the conditions of our previous experiments,\(^7\) both in the boundary layer and in the inviscid core flow. Specifically, for an $M=2.6$ nitrogen flow at a stagnation pressure of $P=1/3$ atm, electrical conductivity of $\sigma=0.1$ mho/m, magnetic field of $B_z=1.5$ T, transverse electric field of $E_z=\pm 300$ V/cm, and the MHD section length of $L=5$ cm, these calculations predict Mach number change by up to $\Delta M=\pm 0.2$. This Mach number change corresponds to a relative static pressure change of $\Delta P/P=\pm 30\%$, which would be easily detectable in the experiment. This suggests that the same experimental apparatus as used in our previous work\(^2\) can be employed to detect the flow acceleration effect.

During the operation of a MHD channel, Joule heat is inevitably generated in addition to the Lorentz force. The ratio of the Joule heat, $j_yE_y$, to the Lorentz force work, $j_yB_zu$, determines the MHD loading parameter, $K=E_y/B_zu$, where $j_y=\sigma E_y$ is the transverse current density and $u$ is the flow...
velocity. Obviously, increasing the electric field at a given conductivity would increase the Lorentz force. However, this would be achieved at the penalty of also increasing the loading parameter, which would mean that a larger fraction of input electrical power would simply heat the gas, without imparting momentum to the flow. In nonequilibrium nitrogen and air plasmas sustained in supersonic flows, the detrimental effect of Joule heating can be significantly reduced due to the well known fact that a major fraction of the electric discharge power at these conditions, up to 98%, goes to vibrational excitation of nitrogen. Since the supersonic flow residence time in the MHD section is quite short, $\tau_{\text{res}} \sim L/u \sim 0.1 \, \text{m}/10^3 \, \text{m/s} \sim 100 \, \mu s$, while vibrational relaxation time of nitrogen at low temperature is $P \tau_{\text{VT}} \sim 1 \, \text{atm} \, \text{s}$, vibrational relaxation simply does not have time to occur. In air, vibrational relaxation of nitrogen in the presence of O atoms generated in the discharge is much faster, $P \tau_{\text{VT}} \sim 10 \, \text{atm} \, \mu s \cdot (n_O/N)^9$ where $n_O/N$ is the O atom mole fraction. However, because the O atom fraction in the discharge is rather small, vibrational relaxation would still remain very slow. For this reason, the energy would remain locked in the nitrogen vibrational mode and Joule heating in the discharge would be greatly reduced. The effective MHD loading parameter at these conditions can be defined as follows:

$$K = \frac{\alpha \cdot j \cdot E_y}{j_y \cdot B \cdot u} = \frac{\alpha \cdot E_y}{B \cdot u},$$

where $\alpha$ is the discharge energy fraction going into Joule heating.

This well known effect provided the rationale for completely neglecting Joule heating in modeling calculations of Ref. 7, as a first approximation. However, this approach is oversimplified, since it is understood that in the actual low-temperature MHD experiments, Joule heating, although significantly reduced, still remained a factor affecting the results. The main objective of the present work is to experimentally study the effect of the Lorentz force on the flow Mach number, determined from static pressure measurements. If the Lorentz force interaction indeed results in significant momentum transfer from the charged species to the entire supersonic flow, the flow static pressure would decrease for both j and B vectors configurations producing an accelerating Lorentz force and increase for the other two configurations producing a retarding Lorentz force. On the other hand, if the electric discharge power at these conditions remains the same, Joule heating would result in a static pressure increase (i.e., Mach number reduction), which would be the same for all four of these cases. In case when both these factors, Lorentz force and Joule heating, generate comparable effects on the flow, the static pressure dependence on the Lorentz force direction should still be apparent.

II. EXPERIMENT

The experiments have been conducted at the supersonic nonequilibrium plasma/MHD wind tunnel facility described in greater detail in Refs. 1 and 2. Briefly, this facility generates stable and diffuse supersonic nonequilibrium plasma flows at $M=3–4$ in a uniform magnetic field up to $B=2 \, \text{T}$, with run durations from tens of seconds to complete steady state. The schematic of the $M=3$ supersonic nozzle and an MHD test section is shown in Fig. 1. An aerodynamically contoured $M=3$ supersonic nozzle made of transparent acrylic plastic is connected to a 2 cm $\times$ 4 cm rectangular cross section test section 12 cm long with an angle step diffuser. The nozzle/test section/diffuser assembly is attached to a vacuum system connected to a 1200 ft$^3$ dump tank pumped out by an Allis-Chalmers 1300 cfm rotary vane vacuum pump. The minimum pressure in the vacuum system sustained by the pump is 35–40 Torr, which necessitates the use of a supersonic diffuser with the nozzle/test section operated at relatively low stagnation and static pressures $(P_0=1/3–1 \, \text{atm}, P_{\text{test}}=7–20 \, \text{Torr})$. The nozzle assembly is equipped with pressure taps measuring plenum pressure as well as static pressures at the beginning and at the end of the test section. The nozzle throat dimensions are 20 mm $\times$ 9.5 mm, which gives a mass flow rate through the test section of $\dot{m}=15 \, \text{g/s}$ at $P_0=1/3 \, \text{atm}$.

Two rectangular electrode blocks 5 cm long are flush mounted in the side test section walls (see Fig. 1). Each electrode block, made of mica ceramic, incorporates a single copper plate electrode 35 mm wide, 45 mm long, and 3 mm thick. The electrode edges are rounded using a Rogowski profile to achieve a more uniform electric field distribution between the electrodes. To accommodate the electrodes, recesses are machined in the ceramic blocks. This creates a 2 mm thick ceramic layer between each electrode and the flow in the test section. On the opposite sides, the electrodes are covered with 2 mm thick acrylic plates. The gaps between the copper electrodes, the ceramic blocks, and the cover acrylic plates are filled with a self-hardening dielectric compound to preclude electrode surface exposure to air and prevent corona formation near the high-voltage electrode surface. Figure 2 shows a photograph of the $M=3$ test sec-
Ionization in the test section is produced using a Chemical Physics Technologies custom designed high-voltage (up to 20–25 kV peak), short pulse duration (≈10–20 ns), high repetition rate (up to 50 kHz) pulsed plasma generator. The plasma generator produces high voltage pulses by compressing 500 V peak, 1 μs long input pulses using several stages of magnetic compression circuits. The use of the insulated gate bipolar transistor switch allows high pulse repetition rates. During the pulser operation, pulse voltage and current are measured using a Tektronix P6015A high voltage probe and a custom-made low-capacitance resistive current probe.

Transverse dc electrical current (sustainer current) in the supersonic flow ionized by the repetitively pulsed discharge is sustained by applying a dc field (up to 500 V/cm) to two 50 mm × 20 mm dc electrode blocks flush mounted in the top and bottom nozzle walls 4 cm apart, perpendicular both to the flow velocity and to the magnetic field direction, as shown in Fig. 1. The applied dc field, which is far too low to produce additional ionization in the flow, except in the cathode layer, is needed to sustain transverse (MHD) current. The dc electrode blocks are made of boron nitride ceramic, with continuous copper electrodes 45 mm long each. The transverse dc field is applied using a DEL 2 kV/3 A power supply operated in a voltage stabilized mode, with a 0.5–1.0 kΩ ballast. Two inductors 1 mH each were placed in the dc circuit in series with both dc electrodes to attenuate high amplitude current pulse propagation into the dc circuit. Current in the dc sustainer circuit is measured using a Tektronix AM503S current probe.

The entire nozzle/test section/diffuser assembly was placed between the poles of a GMW water cooled electromagnet, as shown in Figs. 1 and 2 and attached to a 4 ft long, 6 in. diameter PVC vacuum pipe connected to the vacuum system. To improve the pulsed discharge load impedance matching, the high voltage pulse magnetic compression unit was also mounted inside the magnet, above the test section, which was necessary to nearly completely remove electromagnetic interference from the pulsed discharge. Because of the long line between the pressure tap and the pressure transducer, the actual response time of the transducer may be significantly longer, which required its direct evaluation in the present experiments. As in our previous work,1,2 static pressure was measured by two different combinations of the transverse \( B \) field and the transverse dc electric field directions. Control runs in a cold supersonic flow without plasmas and in an ionized flow without dc electric field applied, i.e., when the time-averaged Lorentz force is zero, have also been conducted. The purpose of this approach was to isolate the MHD effect, which should depend on the Lorentz force di-

Flow temperature downstream of the MHD section was inferred from the nitrogen second positive system emission spectra measured using a Thor Labs 5 m long AFS fiber optic bundle with collimators on each end, and a Princeton Instruments Optical Multichannel Analyzer (OMA) with a 0.5 m monochromator, 1200 g/mm grating blazed at 700 nm, and an ICCD array camera. The collimators were positioned in front of an optical access window in the test section (see Fig. 1), and in front of the slit opening of the spectrometer, respectively. Fiber optic link calibration using a 1.3 mm diameter aperture light source showed the collimator signal collection region to be a cylinder 2–3 mm in diameter and approximately 50 mm long. Therefore these measurements yielded emission spectra averaged along the line of sight passing through the center plane of the flow (see Fig. 1). Rotational temperature of the flow was inferred using a synthetic spectrum with the accurate nitrogen molecular constants,11 rotational line intensities,12 and the experimentally measured slit function of the spectrometer.

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rection, from the polarity-independent effect of Joule heat. The experiments were conducted in nitrogen, dry air, and room air.

III. RESULTS AND DISCUSSION

Figure 3 shows typical single pulse voltage and current oscillograms in an $M=3$ nitrogen flow at $P_0=250$ Torr, $P_{\text{test}}=8.4$ Torr, respectively, at the magnetic field of $B=1.5$ T. The peak voltage and current at these conditions are 13.2 kV and 31.2 A, respectively, with pulse duration (FWHM) of approximately 30 ns. The pulse energy coupled to the flow, calculated from the current and voltage traces at these conditions, was in the range of 1–2 mJ.

Figure 4 shows several voltage pulses generated at the pulse repetition rate of $\nu=40$ kHz, at the same flow conditions. From Fig. 4, it can be seen that at this pulse repetition rate the voltage duty cycle is extremely low, $\sim 30$ ns/25 $\mu$s.

Figure 5 shows dc sustainer Faraday current traces for two different transverse dc electric field polarities, at the conditions of Fig. 4. $U_{\text{PS}}=2$ kV, $R=0.5$ k$\Omega$. Time-averaged currents are 0.95 A (top curve) and 0.86 A (bottom curve). The high reduced electric field during the pulses, $E/N \sim 7 \times 10^{-15}$ V cm$^2$ (70 Td), makes efficient ionization possible by electron impact, the rates of which have strong exponential dependence on $E/N$. On the other hand, the short pulse duration and the low duty cycle greatly improve the plasma stability. Basically, the pulse duration, $\sim 30$ ns, is much shorter than the characteristic time for the ionization instability development, $\sim 10^{-3}$–$10^{-4}$ s. Figure 5 shows dc sustainer current oscillograms in a pulse-ionized $M=3$ nitrogen flow at the conditions of Figs. 3 and 4. In Fig. 5, current traces are shown for dc power supply voltage of $U_{\text{PS}}=2$ kV and ballast resistor of $R=0.5$ k$\Omega$, for two different electric field polarities. Since the dc power supply operates in the voltage stabilized mode, the voltage between the dc electrodes is $U=U_{\text{PS}}-IR$, where $I$ is the sustainer current. In this figure, the current pulses produced during the high voltage pulses are not resolved. It can be seen that after each ionizing pulse the sustainer current reaches approximately $I=2$ A, with the subsequent falloff in a decaying plasma between the pulses, to a minimum value of about $I=0.5$ A. Note that the plasma does not fully decay between the pulses. As can be seen from Fig. 5, the time-averaged currents at these conditions are close, $\langle I \rangle=0.95$ and 0.86 A. In dry air at the same flow and plasma conditions, the time average currents were up to $\langle I \rangle=1.0$–1.3 A. In the entire range of experimental conditions, the discharge plasma appeared uniform and stable, filling the entire volume of the flow in the MHD section. The magnetic field helped stabilize the discharge, dissipating sustainer current oscillations occurring in the absence of the magnetic field. Photographs of the pulser-sustainer plasmas generated in supersonic flows of nitrogen and air can be found in our recent paper. This behavior suggests that the supersonic plasma flow in the MHD section can be analyzed using a quasi-one-dimensional MHD flow model.

The time-averaged dc discharge power added to the flow at these conditions is approximately 1.1–1.5 kW, which in case of instant thermalization would result in the estimated
flow temperature rise of about $\Delta T=70–100$ K, from the baseline core flow temperature at $M=2.9$ of $T=110$ K. However, at the reduced electric field in the sustainer discharge of $E/N=5–6 \times 10^{-16}$ V cm$^2$ (based on the initial core flow temperature), about 90$\%$ of the discharge power in nitrogen and air goes to vibrational excitation of nitrogen, vibrational relaxation rate of which is extremely slow. Basically, the slow vibrational relaxation rate locks up the energy stored in nitrogen vibrations and makes the supersonic flow essentially frozen. Assuming that the rest of the discharge power ($\sim 10\%$) thermalizes, the resultant inviscid core flow temperature rise would be significantly lower, only up to $\sim 10$ K. Note that energy addition to the flow by the repetitively pulsed discharge, based on the measured single pulse energy, 1–2 mJ, is insignificant, 40–80 W at the pulse repetition rate of $\nu=40$ kHz, or only a few per cent of the energy loading by the dc sustainer discharge. These estimates are consistent with the flow temperature measurements. Figure 6 shows two $N_2(\text{C}^3\Pi_u \rightarrow \text{B}^3\Pi_g)$ emission spectra (rotationally unresolved $1 \rightarrow 4$ band) measured in an $M=3$ nitrogen flow ionized by a repetitively pulsed discharge at $\nu=40$ kHz and $B=1.5$ T, (i) without the dc sustainer discharge and (ii) at the highest sustainer discharge power of 1.4 kW, achieved at $U_{ps}=2$ kV, $R=0.5$ k$\Omega$, and $\langle I\rangle=0.9$ A. Note that these spectra are very nearly identical, although the line-of-sight averaging by the fiber optic collimator includes signal contribution from the boundary layers flowing over the dc electrode surfaces (see Fig. 1), where heating by dc discharge is likely to be most intense. The best fit synthetic spectrum, shown in Fig. 7, indicates the line-of-sight averaged temperature of $T=180\pm 20$ K for both these cases (i.e., 40–80 W power added by the pulser alone and 1.5 kW added by pulser and sustainer together). This temperature is somewhat higher than the isentropic flow temperature at $M=2.9$, $T=110$ K. This is most likely due to the line-of-sight averaging across the core flow and two boundary layers on the top and bottom walls of the test section (see Fig. 1). Contribution of warm boundary layer regions (with the recovery temperature of $T_r \approx 270$ K) into the emission signal results in raising the “tail” of the vibrational band, thereby increasing the apparent rotational temperature. This effect has also been observed in our previous work on shock wave control in $M=2$ low-temperature rf plasma flows. We emphasize that the most important result is that the temperatures measured with and without the 1.4 kW dc sustainer discharge turn out to be very close. Note that instant thermalization of the dc sustainer discharge power at these conditions would result in a flow temperature rise of approximately $\Delta T=90$ K. Figure 7, which shows $N_2$ synthetic spectra at $T=100$, 180, and 260 K, illustrates the sensitivity of the temperature inference method used, and demonstrates that a temperature rise of 90 K would be easily detected at the present spectral resolution. Therefore, absence of a detectable temperature rise produced by a 1.4 kW dc sustainer discharge is direct evidence of delayed flow thermalization due to slow vibrational relaxation.

Figure 8 shows normalized test section static pressure traces measured in $M=3$ nitrogen flows at the conditions of Figs. 3–5 with and without Lorentz force applied. The baseline static pressure, measured using the pressure tap downstream of the MHD section shown in Figs. 1 and 2 was $P=8.4$ Torr, which corresponds to the Mach number of $M=2.9$. Turning the pulser on in the presence of magnetic field, without applying transverse dc electric field, i.e., generating ionization in the test section without applying Lorentz force did not produce detectable pressure rise (see Fig. 8). Recent modeling calculations suggest that a large fraction of the nanosecond pulsed discharge voltage drops across a thin cathode sheath, which could result in a localized energy deposition and strong near-wall flow heating. Note that if a significant fraction of the pulsed discharge power ($\sim 40–80$) is indeed deposited into a thin cathode sheath, the resultant rapid localized heating would likely produce local pressure increase. However, the results of static pressure measurements with only the pulsed discharge operating, shown in Figs. 8 and 9 did not show any detectable pressure rise, although the static pressure tap was located on the same wall as the cathode of the pulsed discharge. This

![Intensity vs Wavelength](image1.png)

**FIG. 6.** $N_2(\text{C}^3\Pi_u \rightarrow \text{B}^3\Pi_g)$ emission spectra (1 $\rightarrow$ 4 band) in an $M=3$ nitrogen flow at $P=250$ Torr, $B=1.5$ T, and $\nu=40$ kHz, with and without 1.4 kW dc sustainer discharge. Synthetic spectrum fit indicates rotational temperature of $T=180\pm 20$ K in both cases.

![Intensity vs Wavelength](image2.png)

**FIG. 7.** $N_2(\text{C}^3\Pi_u \rightarrow \text{B}^3\Pi_g)$ synthetic spectra (1 $\rightarrow$ 4 band) at $T=100$, 180, and 160 K, illustrating the temperature inference method sensitivity.
suggests that pulsed discharge energy deposition in the cathode sheath is a relatively minor effect at the present experimental conditions.

In addition to this baseline pressure trace, four pressure traces plotted in Fig. 8 correspond to four possible combinations of the transverse current and the magnetic field vector directions, shown schematically in Fig. 1. Two of these combinations result in accelerating Lorentz force, \( \mathbf{j} \times \mathbf{B} \), while two others produce retarding Lorentz force. In each one of these runs, the pulser was turned on for 0.5 s. It can be seen that in all four cases, generating transverse current in the MHD section results in a static pressure increase. This behavior points to Joule heating of the flow by the transverse dc discharge as one of the sources of the pressure rise. However, for both \( \mathbf{j} \) and \( \mathbf{B} \) vector combinations corresponding to the accelerating Lorentz force the pressure rise, 5%–7%, is noticeably lower than for both retarding Lorentz force combinations, 18%–21%. The dependence of the static pressure rise on the Lorentz force polarity suggests that the pressure and the flow Mach number may also be affected by the MHD force interaction. Similar results were obtained in a dry air flow at the same flow conditions, see Fig. 9, 5%–7% for the accelerating Lorentz force and 17%–20% for the retarding Lorentz force.

Control runs in nitrogen and dry air have been made with the magnetic field turned off, at \( \mathbf{B} = 0 \). In the absence of the magnetic field, sustainer discharge voltage had to be reduced to \( U_{PS} = 1 \) kV to prevent sustainer current oscillations and instability development. As a result, the discharge power decreased from about 1.5 kW (see Fig. 5) to about 0.5 kW. The sustainer current at these conditions, \( \langle I \rangle = 0.60–0.65 \) A, was comparable to the current at \( B = 1.5 \) T, \( \langle I \rangle = 0.86–0.95 \) A (see Fig. 5). In this case, no pressure difference was detected between two dc discharge polarities, the pressure rise being about 3% in both cases (see Fig. 10). This provides additional evidence that the static pressure difference detected at \( B = 1.5 \) T and shown in Figs. 8 and 9 is indeed due to the Lorentz force interaction.

To analyze the results of static pressure measurements in the presence of the Lorentz force and Joule heat, we have used quasi-one-dimensional MHD flow equations,\(^{16}\)

\[
\frac{dp}{dx} = \frac{1}{M^2 - 1} \left[ - \left( \gamma - 1 \right) M^2 + 1 \right] \cdot \frac{\left( \gamma - 1 \right) M}{a} \cdot \dot{Q},
\]

\[
\frac{dM}{dx} = \frac{1}{ap} \cdot \frac{1}{M^2 - 1} \left[ \left( \gamma + \frac{1}{2} \right) \gamma \right] \cdot \frac{u}{1 + \frac{\gamma - 1}{2} M^2} \cdot F
\]

\[
- \frac{\gamma - 1}{2} \gamma \cdot \left( \gamma M^2 + 1 \right) \cdot \dot{Q},
\]

where \( \gamma \) is the adiabatic index. These equations can be solved numerically, and the resulting pressure traces are shown in Figs. 8 and 9. The legend is the same as in Fig. 6.

FIG. 8. Normalized static pressure traces at the conditions of Figs. 3–5. Lorentz force is applied for 0.5 s duration. Two pressure traces corresponding to two combinations of current (\( \mathbf{j} \)) and magnetic field (\( \mathbf{B} \)) vectors are shown for both accelerating and retarding force directions.

FIG. 9. Normalized static pressure traces in \( M = 3 \) dry air flows at \( P_0 = 250 \) Torr, \( P_{test} = 8.7 \) Torr, \( B = 1.5 \) T, \( \nu = 40 \) kHz, \( U_{PS} = 2 \) kV, \( R = 1.0 \) kΩ. Lorentz force is applied for 0.5 s duration. The legend is the same as in Fig. 8.

FIG. 10. Normalized static pressure traces in \( M = 3 \) nitrogen flows at \( P_0 = 250 \) Torr, \( U_{PS} = 1 \) kV, \( R = 0.5 \) kΩ, without magnetic field. Two pressure traces corresponding to two different transverse dc electric field polarities are shown.
\[
\frac{du}{dx} = \frac{u}{p} - \frac{1}{M^2 - 1} \left( F - \frac{\gamma - 1}{\gamma u} \cdot \dot{Q} \right),
\]
(4)

\[
\frac{dT}{dx} = \frac{T}{p} - \frac{\gamma - 1}{\gamma} \cdot \frac{1}{M^2 - 1} \left( - \gamma M^2 \cdot F + \frac{\gamma M^2 - 1}{u} \cdot \dot{Q} \right),
\]
(5)

where
\[
F = j_y B_z \equiv \frac{IB}{A},
\]
(6)

\[
\dot{Q} = \alpha \cdot j_y E_y \equiv \frac{I (U_{PS} - IR)}{Ah},
\]
(7)

are the Lorentz force and the Joule heat per unit volume, respectively. In Eqs. (6) and (7), I is the sustainer current, \(U_{PS}\) is the dc voltage, \(R\) is the ballast resistance, \(A\) is the dc electrode surface area, \(h\) is the distance between the dc electrodes, and \(\alpha\) is the discharge power fraction going to Joule heating (effective Joule heating factor). Note that for small values of the MHD interaction parameter, \(\eta\),
\[
\eta = \frac{|j_y B_z| L}{\rho u_i^2},
\]
(8)

the right-hand sides of Eqs. (2)–(5) are nearly constant, and they can be integrated analytically. In Eq. (8), \(L\) is the length of the MHD section. Indeed, at the conditions of the present experiments, \(U_{PS} = 2\, \text{kV}, I = 1.0\, \text{A}, R = 0.5\, \Omega, B = 1.5\, \text{T}, A = 9\, \text{cm}^2, L = 4.5\, \text{cm}, \rho = 0.03\, \text{kg/m}^3, \) and \(u_i \approx 600\, \text{m/s}, \) the interaction parameter is quite low, \(\eta = 7 \cdot 10^{-3}\). Then integrating Eq. (2) gives the following expressions for the pressure rise difference between the retarding and the accelerating Lorentz force cases,
\[
\Delta p_R - \Delta p_A \equiv 2 \cdot \frac{\gamma - 1}{M^2 - 1} \cdot j_y B_z L,
\]
(9)

and for the effective Joule heating factor,
\[
\alpha \equiv \frac{\Delta p_A + \Delta p_R}{2 \cdot \frac{\gamma - 1}{\gamma - 1} M \cdot j_y B_z L}.
\]
(10)

For the baseline conditions, \(p = 8.5\, \text{Torr}, T = 110\, \text{K}, M = 2.9, \gamma = 1.4,\) Eq. (9) gives \((\Delta p_R - \Delta p_A)/p \approx 0.08\) for \(I = 1\, \text{A}\.\) Note that the estimated pressure difference is consistent with the experimental results for nitrogen and air shown in Figs. 8 and 9. Using Eq. (10) with the results of Fig. 8, the effective Joule heating factor is \(\alpha = 0.11 \pm 0.02\). The effective Joule heating factor inferred from the static pressure rise measurements is in good agreement with the results of Boltzmann equation solution,\(^{15}\) \(\alpha = 0.09\) in nitrogen at the reduced electric field of \(E/N = 6 \cdot 10^{16}\, \text{V/cm}^2\) and \(\alpha = 0.10\) in air at \(E/N = 5 \cdot 10^{16}\, \text{V/cm}^2\). At these conditions, the effective MHD loading parameter (the ratio of the Joule heating and the Lorentz force work), given by Eq. (1), is \(K \approx 4\). Therefore, this analysis suggests that the observed static pressure difference between the accelerating and the retarding Lorentz force runs is indeed due to the MHD force interaction, superimposed on the pressure rise due to Joule heating of the flow in the discharge.

The rate of the flow velocity change due to MHD interaction, predicted by Eq. (4), can be also estimated from simple analysis of momentum transfer from the charged particles (electrons and ions) to the neutrals by collisions. Indeed, the Lorentz force applied to the plasma as a whole is balanced by the collision drag force,
\[
e(n, \mu_e + n, \mu_i) E_y B_z = (n_m \nu_m + n_i \nu_i)(u_p - u).
\]
(11)

Note that the Coulomb forces on the electrons and the ions produced by the Hall (polarization) field cancel out. In Eq. (11), \(n_e\) and \(n_i\) are electron and ion number densities, respectively, \(m_e\) and \(m_i\) are their masses, \(\mu_e\) and \(\mu_i\) are their mobilities, \(\nu_m\) and \(\nu_i\) are electron-neutral and ion-neutral collision frequencies, and \(u_p - u\) is the plasma velocity relative to the flow (ion slip velocity\(^{16}\)). Assuming that charge separation in the plasma due to the Hall effect is small, so that \(n_e = n_i\) and using \(\mu_e = e/m_e \nu_m\) and \(\mu_i = e/m_i \nu_i\), we have
\[
e(\mu_e + \mu_i) E_y B_z = \left( \frac{1}{\mu_e} + \frac{1}{\mu_i} \right) (u_p - u).
\]
(12)

Finally, remembering that \(\mu_e > \mu_i\), the ion slip velocity is
\[
u_i / u \approx \mu_e \mu_i E_y B_z.
\]
(13)

Equation (13), obtained in Ref. 19 using a more detailed and rigorous analysis, is in good agreement with recent measurements of the ion slip velocity (near-surface constrained discharge filament velocity) in a weakly ionized low-temperature \(M = 3\) flow at \(B = 2\, \text{T}.\)\(^{6}\) Since the neutral velocity changes in an ion-neutral collision and in a electron-neutral collision are \((m_e/m_i)(u_p - u)\approx (u_p - u)\), respectively, the net rate of the neutral flow velocity change is
\[
du / dt = u_n \cdot n_e (u_p - u) = \frac{j_y B_z}{\rho} = \frac{F}{\rho}.
\]
(14)

Finally, remembering that \(\rho = \gamma \rho_m / u^2\), we obtain
\[
du / dx = u \cdot \frac{F}{\gamma \rho_m}.
\]
(15)

which at \(M^2 > 1\) and in the absence of Joule heat is consistent with Eq. (4). This simple estimate, along with the ion slip velocity measurements,\(^6\) demonstrates that phenomenological MHD flow equations, Eqs. (2)–(5), are consistent with the plasma behavior on the microscopic level.

To estimate the effect of the rate of vibrational relaxation of nitrogen on the effective Joule heating factor, \(\alpha\), one series of experiments was done in room air. In this case, two additional factors change the sustainer discharge characteristics considerably. First, energy added to the vibrational mode of nitrogen by the sustainer discharge thermalizes more rapidly due to fast vibrational relaxation of nitrogen on water vapor, \(P_i = 10\, \text{atm} \cdot \mu_s\) at room temperature.\(^{20}\) Second, electrons in the plasma rapidly attach to oxygen in three-body collisions with water molecules.
(16)

$$e + O_2 + H_2O \rightarrow O_2^- + H_2O,$$

with a near gas kinetic rate, \(k = 1.4 \cdot 10^{-29} \text{ cm}^6/\text{s}.\) The latter effect is clearly evident in Fig. 11, which shows sustainer discharge currents in \(M=3\) dry air and room air flows at the same conditions, \(P_0=250\) Torr, \(P_{\text{test}}=8.7\) Torr, \(B=1.5\) T, \(U_{\text{PS}}=2\) kV, and \(R=1.0\) k\(\Omega\). It can be seen that adding water vapor reduces the current decay time (plasma lifetime) by about an order of magnitude, from about 25 \(\mu\)s to about 2–3 \(\mu\)s. Because of this, the time-averaged current in Fig. 11 also drops by a factor of 10, from \(\langle I \rangle = 0.51\) A to \(\langle I \rangle = 0.052\) A, which suggests that the Lorentz force effect in room air flows would be negligibly small. Indeed, Fig. 12 shows essentially no difference in static pressures rise for the accelerating and retarding Lorentz force directions in room air flows at these conditions. However, from Fig. 12 it can be seen that the static pressure rise due to Joule heating in room air, 6±2%, is comparable to the pressure rise in dry air at the same flow conditions (see Fig. 9), in spite of an order of magnitude difference in the sustainer discharge current and power. Using Eq. (10) with the results of Fig. 12 gives an estimate of the effective Joule heating factor in room air, \(\alpha = 0.4 \pm 0.15\). This result shows that adding water vapor substantially accelerates the rate of Joule heating in supersonic nonequilibrium plasma flows, most likely due to accelerated vibrational relaxation of nitrogen.

The characteristic time for the flow static pressure change due to both MHD forcing and Joule heating of the supersonic core flow should be comparable with the flow residence time in the discharge section, \(\sim 100\) \(\mu\)s. However, the pressure rise/fall time measured in the present experiments is much longer, \(\sim 0.2\) s (see Figs. 8–10 and 12), which is about an order of magnitude longer than the time resolution of the data acquisition system used, about 15 ms. Clearly, the measured rise/fall time is affected by an additional factor. Varying the length of the \(\frac{1}{4}\) in. diameter plastic line connecting the wall static pressure tap and the pressure transducer showed that it is in fact the long line that controls the pressure measurement system response time. Indeed, Fig. 13 shows the normalized static pressure signals measured at the same conditions, in an \(M=3\) nitrogen flow with a retarding Lorentz force applied, for two different line lengths, 1.9 m and 4.4 m. It can be seen that increasing the line length also increased the signal response time from about 0.2 s to about 0.5 s, without changing the steady-state pressure value. In the present experiments, removing the line and placing the pressure transducer near the static pressure tap was not feasible because of strong electromagnetic interference of the pulsed discharge with the transducer.

Figure 14 compares static pressure measurements with the results of numerical integration of Eqs. (2)–(5), for three different values of the effective Joule heating parameter, \(\alpha = 0\) (no Joule heating), \(\alpha = 0.05\), and \(\alpha = 0.10\). Experimental points in Fig. 14 are obtained by measuring the static pressure 0.2 s after turning the pulser on, approximately at the moment when the pressure reached near steady state. In Fig.
14, positive values of the current correspond to the accelerating Lorentz force. It can be seen that the results of calculations for $\alpha=0.10$ are in good agreement with the experimental data. Figure 14 also illustrates how Joule heating superimposed over Lorentz force affects the static pressure change. Specifically, while the calculations at $\alpha=0$ (no Joule heating) predict static pressure reduction (i.e., Mach number increase) for the accelerating MHD force and static pressure increase (i.e., Mach number reduction) for the retarding MHD force, Joule heating results in static pressure rise in both cases. However, the predicted static pressure increase is always higher for the retarding MHD force, which was observed in the present experiments.

Figure 15 plots the calculated flow Mach number at the conditions of Fig. 14. Comparing the results shown in Figs. 14 and 15 at $\alpha=0.10$, it can be seen that at $I=\pm 1$ A flipping the Lorentz force direction from accelerating to retarding results in a Mach number change from $M=2.77$ to $2.64$ ($\Delta M=0.13$). Since the baseline Mach number is $M=2.89$, it is apparent that in both these cases the combined effect of Lorentz force and Joule heating results in flow deceleration. However, as expected, flow deceleration is more pronounced for the retarding Lorentz force. From Fig. 15, one can also see that in the absence of Joule heating the Mach number change at $I=\pm 1$ A would be from $M=2.96$ to $2.83$ (at the same baseline Mach number of $M=2.89$), $\Delta M=0.13$. This Mach number change is lower than predicted by the 3D compressible MHD Navier-Stokes calculations, $\Delta M=0.4$ at the baseline Mach number of $M=2.6$. However, in these calculations the MHD current was assumed to be $I=3$ A, which is approximately a factor of three higher than has been achieved in the present experiments. Raising transverse current in the quasi-one-dimensional model of Eqs. (2)–(5) up to $I=3$ A results in the increase of the Mach number change, $\Delta M=0.42$, which is very close to the result obtained in Ref. 7.

Figure 16 summarizes the results for the normalized static pressure difference for two Lorentz force directions, $(\Delta p_R-\Delta p_A)/p$, as a function of the transverse sustainer current obtained in $M=3$ nitrogen and dry air flows. It can be seen that the measured relative pressure change increases nearly proportional to the current and reaches about 13% at $|I|=1.2–1.3$ A. This behavior is in good agreement with the quasi-one-dimensional MHD theory, both an approximate analytic solution, Eq. (9), and numerical integration of coupled Eqs. (2)–(5). We conclude that the dependence of the static pressure change on the Lorentz force magnitude and polarity, which is consistent with the results of the quasi-one-dimensional MHD flow analysis, conclusively demonstrates supersonic flow deceleration by the Lorentz force. We emphasize that this effect could be detected only because the Joule heating factor in nitrogen and in dry air is small, $\alpha=0.1$. If this were not the case, at low electrical conductivities achieved at the present experimental conditions the MHD effect would be overshadowed by Joule heating of the flow. To the best of our knowledge, this is the first time this effect was experi-
mentally demonstrated in cold supersonic gas flows. This result, however, does not show feasibility of large-scale MHD deceleration of supersonic flows, discussed in Ref. 6. This would require analysis of other critical technical issues, such as sustaining magnetic field and low energy cost external ionization over large volumes of the flow, as well as boundary layer separation in a decelerating flow.

Demonstration of net MHD acceleration of the flow, when the Mach number increase and the static pressure is reduced, would require reducing Joule heating, \( Q \), while keeping the Lorentz force, \( F \), the same. From Eqs. (2) and (3), it can be seen that this is equivalent to reducing the loading parameter, determined by Eq. (1), to \( K \sim 1 \). Since at the present experimental conditions the loading parameter is \( K \sim 4 \) (at \( \alpha = 0.1 \)), this suggests that net flow acceleration could be achieved if either the effective Joule heating factor or transverse electric field are reduced by a factor of 4, down to \( \alpha = 0.025 \) or \( E_y = 100 \text{ V/cm} \), respectively, or if the magnetic field is increased by a factor of 4, up to \( B = 6 \text{ T} \). Note that keeping the Lorentz force the same while reducing the electric field can be done only if the effective electrical conductivity of the flow, \( \sigma \), is increased, so that the same transverse current, \( J_x = \alpha E_y \), would be sustained at a lower transverse electric field. Therefore reducing the electric field by a factor of 4 would require quadrupling the conductivity.

Although the Mach number change inferred from the present static pressure measurements is consistent with the predictions of the three-dimensional compressible MHD Navier-Stokes calculations,\(^7\) these calculations were conducted neglecting Joule heating of the flow in the discharge and forces on the plasma (including ion slip). More accurate modeling calculations, taking into account dynamics of the plasma and the experimentally evaluated discharge energy fraction going into Joule heating, would provide better insight into MHD acceleration and Joule heating of both inviscid core flow and boundary layers.

IV. SUMMARY

The paper presents results of cold MHD flow deceleration experiments using repetitively pulsed, short pulse duration, high voltage discharge to produce ionization in \( M = 3 \) nitrogen and air flows in the presence of transverse dc electric field and transverse magnetic field. MHD effect on the flow is detected from the flow static pressure measurements. Retarding Lorentz force applied to the flow produces a static pressure increase of up to 17%–20%, while accelerating force of the same magnitude applied to the same flow results in static pressure increase of up to 5%–7%. The effect is produced for two possible combinations of the magnetic field and transverse current directions producing the same Lorentz force direction (both for accelerating and retarding force). This demonstrates that the observed static pressure change is indeed due to the MHD interaction, and not due to Joule heating of the flow in the crossed discharge. No discharge polarity effect on the static pressure was detected in the absence of the magnetic field. The measured static pressure changes are compared with modeling calculations using quasi-one-dimensional MHD flow equations. The fraction of the discharge input power going into Joule heat in nitrogen and dry air, \( \alpha = 0.1 \), has been inferred from the present experiments, and used as one of the input parameters in the MHD flow model. This fraction is low, primarily because most of the discharge power remains frozen in the vibrational energy mode of nitrogen, and increases to \( \alpha = 0.4 \pm 0.15 \) in room air because of rapid nitrogen relaxation on water vapor. Comparison of the experimental results with the modeling calculations shows that the retarding Lorentz force increases the static pressure rise produced by Joule heating of the flow in the discharge, while the accelerating Lorentz force reduces the pressure rise. This result provides first direct evidence of cold supersonic flow deceleration by Lorentz force.

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