Emission and shock visualization in nonequilibrium nitrogen afterglow plasma

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Kinetic modeling of propagating and stationary normal shocks in nonequilibrium nitrogen afterglow plasma is used to simulate the results of shock emission measurements in nitrogen afterglow. Emission intensity overshoot behind the shock predicted by the model is in satisfactory agreement with the experimental results and is consistent with previous analytic estimates. The model demonstrates that the first and the second positive band emission overshoot behind the shock are produced by energy transfer processes among the triplet electronic states of nitrogen generated in the electric discharge. On the other hand, charge separation and ambipolar electric field produced across the shock layer do not result in electron heating and additional electron impact excitation of electronic states. The calculations show that emission overshoot makes possible accurate detection of a stationary shock layer in supersonic flowing afterglow experiments. © 2007 American Institute of Physics. [DOI: 10.1063/1.2798984]

I. INTRODUCTION

The effect of shock waves on visible emission from nonequilibrium plasmas has recently attracted considerable attention. Experiments in supersonic flowing afterglow plasmas showed that stationary oblique and bow shocks significantly enhance emission from the plasma, which results in shock visualization. This effect was detected both for the first positive and the second positive band systems of nitrogen. Recent time-resolved measurements of visible emission produced by a propagating shock wave in a low-pressure nitrogen afterglow plasma also showed significant emission enhancement by the shock, exceeding both steady-state values in a dc glow discharge plasma and Rankine-Hugoniot normal shock density jump conditions.

Mechanism of plasma emission enhancement by a shock wave is important both for understanding kinetics of non-equilibrium afterglow plasmas and for using it for high-speed flow visualization. One of the key technical issues is to what extent the emission enhancement is due to simple flow compression by the shock and whether it is affected by nonlinear energy pooling processes among electronically excited molecules generated in the plasma. The other question is whether emission enhancement behind the shock may be related to charge separation and resultant local electric field in the vicinity of the shock layer. In Ref. 4, it was estimated that the characteristic time for energy pooling reactions, such as

\[ N_2(A^3\Sigma) + N_2(A^3\Sigma) \rightarrow \begin{cases} \text{N}_2(B^3\Pi) + \text{N}_2(X^1\Sigma) \\ N_2(C^3\Pi) + \text{N}_2(X^1\Sigma), \end{cases} \]

\[ \sim 300-600 \mu s, \] is much longer than the measured emission rise time behind the shock, \(\sim 2 \mu s\). On the other hand, it was also pointed out that charge separation by the shock and double layer formation in the vicinity of the shock layer are unlikely to be responsible for the emission enhancement. Indeed, the ambipolar electric field in the double layer is directed against the electron flux and therefore acts to reduce the electron energy. Therefore, emission enhancement by electron impact, with electrons heated up in the double layer, does not appear feasible. However, Ref. 4 leaves the door open to a possibility of emission enhancement by electron impact excitation, due to a nonlocal electron energy distribution that might be produced in a strong double layer.

A recent comment on the interpretation of experimental results of Ref. 4 points out that the rise time for the second positive system and for the first positive system emission, \(N_2(C^3\Pi \rightarrow B^3\Pi)\) and \(N_2(B^3\Pi \rightarrow A^3\Sigma)\), behind the shock is, in fact, consistent with the characteristic time for the population of \(B^3\Pi\) and \(C^3\Pi\) states by the energy pooling processes of Eq. (1) and for \(B^3\Pi\) state excitation in collisions of molecules in the \(A^3\Sigma\) state with vibrationally excited molecules in the ground electronic state,

\[ N_2(A^3\Sigma) + N_2(X^1\Sigma,v = 5-14) \rightarrow N_2(B^3\Pi) + N_2(X^1\Sigma). \]

This occurs because populations of radiating \(B^3\Pi\) and \(C^3\Pi\) states in the afterglow are much lower than that of the metastable \(A^3\Sigma\) state. The estimates of Ref. 6 suggest that emission enhancement behind the shock can be explained by collisional and radiative energy transfer among the triplet states of nitrogen and does not require additional electron impact excitation in the double layer.

The objectives of the present paper is (i) to elucidate the kinetic mechanism of emission enhancement by a propagating shock in a nitrogen afterglow plasma and (ii) to provide insight into the mechanism of stationary shock visualization in supersonic flowing nitrogen afterglow plasma.
II. KINETIC MODEL

In the present paper, glow discharge in nitrogen and decaying afterglow plasma are modeled in a quasi-one-dimensional approximation, with one spatial dimension along the discharge tube axis (i.e., in the direction of shock propagation). The model incorporates a set of unsteady one-dimensional compressible Navier-Stokes flow equations (continuity, motion, and energy equations) for nitrogen molecules in the ground electronic state, Eq. (A1), as well as for ions and electrons, Eqs. (A2) and (A3), and the Poisson equation for the electric field, Eq. (A4).\(^8\) This set of equations is coupled with the equations for the number densities of electronically excited nitrogen molecules, \(N_2(A\,^3\Sigma),\) \(N_2(B\,^3\Pi),\) \(N_2(C\,^3\Pi),\) and \(N_2(a\,^1\Sigma),\) Eq. (A5), and nitrogen atoms, Eq. (A6). The entire set of equations is listed in the Appendix. Transport processes in the transverse (radial) direction, such as diffusion, ambipolar diffusion, and heat conduction, are modeled using characteristic times for these processes in the equations (see Appendix). After the discharge reaches steady state, the external electric field is turned off, and the shock wave is launched after a delay time ranging from 1 to 300 \(\mu s\). The normal shock is initiated by creating a triangular shape pressure, density, and temperature perturbations, with the amplitudes determined by the Rankine-Hugoniot shock jump conditions. The time-dependent emission from the afterglow plasma is monitored at a fixed location 5 mm away from the initial perturbation.

To model spatial distribution of plasma emission intensity behind a stationary shock in a supersonic flowing afterglow, the approach was modified. In this case, the discharge region was located 5 mm upstream of the normal shock standing in the flow, and the modeling calculations were done in a reference frame attached to the shock, until steady state was reached.

The kinetic model used in the present paper incorporates processes of vibrational excitation, electronic excitation, dissociation, and ionization of nitrogen by electron impact, as well as collisional and radiative processes of population and quenching of electronically excited states of nitrogen [\(N_2(A\,^3\Sigma),\) \(N_2(B\,^3\Pi),\) \(N_2(C\,^3\Pi),\) and \(N_2(a\,^1\Sigma)\)]. The electron impact excitation rates as functions of the reduced electric field, \(E/N,\) have been calculated separately by solving two-term expansion Boltzmann equation for plasma electrons.\(^9\) To reduce the number of coupled partial differential equations solved, the steady-state vibrational level populations of the ground electronic state, \(N_2(X\,^1\Sigma, v),\) are evaluated using analytical theory of vibrational energy transfer at the conditions of strong vibrational disequilibrium.\(^10\) In this case, relative populations of the low vibrational levels,

\[
f_v = \left\{ \begin{array}{ll}
    f_0 \exp \left( \frac{-E_v}{T_v} + \frac{\Delta E_v (v-1)}{T} \right), & v \leq v_w \\
    f_w \frac{v_0 + 1}{v + 1}, & v > v_w,
\end{array} \right.
\]

(3)

are independent of the rates of vibration-translation (V-T) processes since their rates are slower than the rates of the vibration-vibration (V-V) processes by several orders of magnitude.\(^11,12\) In the Treanor/V-V pumped plateau distribution of Eq. (3), \(E_v = \omega_v (1-2x_v), \Delta E_v = \omega_v x_v, \) \(T_v = E_v / \ln (f_0 / f_1)\) is the vibrational temperature of the first level, \(v_w = 1/2 + T/2x(T)\), and \(f_0 = f_0 \exp \left( -E_v / T v T_0 \right).\)\(^10\) The first level vibrational temperature is determined from the rate of vibrational excitation by electron impact being equal to the net rate of vibrational energy relaxation in the V-V processes exchange,\(^10,13\)

\[
\sum f_v k_{v0-v} \frac{n_v}{N} = 6 \frac{Q_w \Delta E_v}{e T \delta_v} (v_w + 1)^2 f_w^2.
\]

(4)

In Eq. (4), \(E_v = \omega_v (1-x_v (v+1)), \) \(k_{v0-v}\), are the rates of vibrational excitation by electron impact, \(n_v / N\) is the ionization fraction, \(e\) is the natural logarithm base, and \(Q_w\) and \(\delta_v\) are the parameters in the expression for the V-V rates,\(^11\)

\[
k_v (v + 1, w \rightarrow v, w + 1) = (v + 1)(w + 1) Q_w \frac{y^3 - y}{2} \exp (-\delta_v |v - w|).
\]

(5)

The analytic population distribution of Eq. (3) is valid if the V-T rates are much slower compared to the V-V rates. Previously, we have shown that the population distribution of Eq. (3) is in good agreement with the master equation modeling calculations.\(^13\) The present approach assumes that the vibrational level populations do not significantly change over the delay time between turning the electric field off and shock arrival, i.e., that the vibrational energy relaxation rate is slow.

The rates of collisional and radiative excitation and quenching of the excited electronic states, energy pooling processes, nitrogen atom recombination, associative ionization, and electron-ion recombination have been taken from the \(N_2\) glow discharge model of Ref. 14, which has been validated by comparing with experimental measurements.\(^15-17\) The list of processes and rates used, taken from Refs. 11, 14, and 18-22, is summarized in Table I. In the present model, it was assumed that the dominant ion in the afterglow plasma is \(N_4^+\), since the characteristic time for the ion conversion,

\[
N_2^+ + N_2 + M \rightarrow N_4^+ + M,
\]

(6)

at \(P \sim 1\) torr is relatively short, \(\tau_{\text{comb}} \sim 10\) \(\mu s.\)\(^14\) For this reason, we used the electron-ion dissociative recombination rate for \(N_4^+\) ions.

According to the results of modeling calculations incorporating state-specific vibrational kinetics,\(^14\) nitrogen dissociation from high vibrational levels of the ground electronic state, \(N_2(X\,^1\Sigma, v \sim 45),\) is insignificant compared to the dissociation by electron impact, due to rapid V-T relaxation of these levels in collisions with N atoms. For this reason, vibrationally induced dissociation is not incorporated in the present model. Note that the accuracy of the available theoretical V-T rates for \(N_2\) (Ref. 23) cannot be evaluated due to lack of experimental data.

Note that a large difference between the rates of electron and ion diffusion introduces significant stiffness in the sys-
tem of equations listed in the Appendix. The equations are integrated by a standard solver for stiff partial differential equations, PDECOL,24 using 1000 mesh points equally spaced across the integration domain.

### III. RESULTS AND DISCUSSION

First series of modeling calculations was done for the experimental conditions of Ref. 4, glow discharge in nitrogen in a 1.5 cm radius Pyrex tube at $P=0.75$ torr, $E = 24.5$ V/cm ($E/N \approx 10 \times 10^{-16}$ V cm$^2$), and $J=20$ mA ($n_e = 1.7 \times 10^9$ cm$^{-3}$). Figure 1 shows calculated steady-state species populations in the discharge as functions of the current density. At the experimental conditions modeled in the present paper, the current density is $j=2.83$ mA/cm$^2$. From Fig. 1, it can be seen that the number densities of the radiating $B ^3\Pi$ and $C ^3\Pi$ states in the entire range of current densities are much lower than that of the $A ^3\Sigma$ state. One can also see that the present model underestimates the number density of $N_2(A ^3\Sigma)$ molecules compared to the steady-state experimental data,16 (by about a factor of 2.5 at the present conditions). Changing the relative contributions of the two channels of the $N_2(B ^3\Pi)$ state quenching, $N_2(B) + N_2(X) \rightarrow N_2(A) + N_2(X)$ and $N_2(B) + N_2(X) \rightarrow N_2(X) + N_2(X)$, from their base line values of 2/3 and 1/3 (Ref. 19) to 0.95 and 0.05 (Ref. 14) doubles the $N_2(A ^3\Sigma)$ number density and improves agreement with the steady-state experimental results. However, this adjustment makes agreement with the time-resolved afterglow emission data, shown in Fig. 2, considerably worse. In particular, the predicted second positive emission drop, immediately after the discharge is turned off, is reduced from a factor of 5 to a factor of 2, at variance with the time-resolved measurements of Ref. 4. For this reason, we used the rate coefficients of the $N_2(B ^3\Pi)$ state quenching recommended in Ref. 19. The steady-state vibrational temperature at $E/N=10.3 \times 10^{-16}$ V cm$^2$ and $j=2.83$ mA/cm$^2$, determined from Eq. (4), is $T=3200$ K ($v_{ii}=9$, $f_{ii}=8 \times 10^{-3}$). After the discharge reached the steady state, the electric field was suddenly turned off, and a shock was launched after a delay time ranging from 1 to 300 $\mu$s. The shock Mach number was varied from $M=1.1$ to $M=3.8$.

Figure 2 compares the experimental first positive and the second positive system emission of nitrogen, $N_2(B ^3\Pi$
duced by a passing shock, with the predictions of the process of Eq. (1) after the field is turned off, detected in the experiment. A gradual reduction of the energy pooling mechanism of Eq. (1) is reproduced less accurately. The loss in the energy pooling process of Eq. (1) is very weak and is not shown in Fig. 2. Note that this calculation gives only an estimate of the V-T relaxation effect, for the theoretically predicted V-T rates. Incorporating both a propagating shock and state-specific vibrational kinetics into the present model would require solving 50 coupled partial differential equations over ~1000 mesh points, which would be a computationally demanding problem.

Figure 2 also shows that the model correctly predicts rapid emission intensity rise behind the passing shock, both for the first positive and the second positive systems. The second positive system emission rise is mainly due to the C state population by the energy pooling process of Eq. (1), in collisions of two N2(A 3Σ) molecules compressed by the shock. The characteristic second positive system emission rise time, \( \tau_{\text{2nd}} \approx k_{\text{pool}} n_A^2 / n_C \), is very short. This occurs due to a much higher A 3Σ state number density at steady state, \( n_A = 1.3 \times 10^{12} \text{ cm}^{-3} \), compared to that of the C 3Π state, \( n_C = 4 \times 10^{10} \text{ cm}^{-3} \) (see Fig. 1). The first positive emission rise is mainly due to the rate at \( t=10–100 \mu s \), most likely because the characteristic time of vibrational energy relaxation of N2(X,v) by V-V energy transfer,

\[
\tau_{\text{v-v}} = \left[ \frac{6Q_{10} N_{X} (v_{X} + 1)^2 f_{10}}{eT_{v} \delta_{v}} \right]^{-1},
\]

\( \tau_{\text{v-v}} \approx 500 \mu s \) at the present conditions, is underestimated. The vibrational relaxation time is likely to be somewhat shorter due to the contribution of V-T relaxation on N atoms, which is not incorporated in the present model. Faster vibrational relaxation would decrease the amount of N2(X,v > 5) molecules and would therefore reduce the rate of N2(B 3Π) population in the process of Eq. (2), thereby bringing net B 3Π decay rate closer to the experimental result (see Fig. 2).

To estimate the effect of nitrogen V-T relaxation on the afterglow emission, N2 vibrational energy after the discharge was assumed to relax with the relaxation time of NV = 240 \( \mu s \). This value of V-T relaxation time was obtained using a room temperature 1 – 0 V-T relaxation rate,\(^{23} k_{10} = 4.2 \times 10^{-11} \text{ cm}^3/\text{s} \). From Fig. 2, it can be seen that with this process taken into account, the first positive system emission decay becomes more rapid and somewhat closer to the experimental trace. The effect on the second positive emission is very weak and is not shown in Fig. 2. The dash-dotted curve in Fig. 2 shows predicted the time-dependent emissions N2(B-A) and calculated \( N_2(B-A) \) and \( N_2(C-B) \) at shock Mach numbers of 3.4.

\( k_{10} = 1.5 \times 10^{-10} \text{ cm}^3/\text{s} \) is the rate coefficient of reaction of Eq. (1). It is not shown in Fig. 2 because it is much higher than the \( k_{10} = 1.5 \times 10^{-10} \text{ cm}^3/\text{s} \) at the same temperature. This occurs due to a much higher A 3Σ state number density at steady state, \( n_A = 1.3 \times 10^{12} \text{ cm}^{-3} \), compared to that of the C 3Π state, \( n_C = 4 \times 10^{10} \text{ cm}^{-3} \) (see Fig. 1). The first positive emission rise is mainly due to the vibrational relaxation on N atoms, which is not incorporated in the present model.
states behind the shock are spontaneous radiative decay, scaled for the relative populations of \( \frac{1}{nX} \) respectively, immediately before the shock arrival shoots scales approximately as \( \frac{1}{nX} \). The dominant quenching processes of \( C^3\Pi \) and \( B^3\Pi \) states behind the shock are spontaneous radiative decay,

\[
N_2(C^3\Pi) \rightarrow N_2(B^3\Pi) + h\nu,
\]
and collisional quenching by the ground state molecules,

\[
N_2(B^3\Pi) + N_2(X^1\Sigma) \rightarrow \begin{cases} N_2(A^3\Sigma) + N_2(X^1\Sigma) \\ N_2(X^1\Sigma) + N_2(X^1\Sigma), \end{cases}
\]
respectively. Note that the first positive emission overshoot is somewhat overpredicted by the model (a factor of 1.9 greater than the steady-state emission versus approximately a factor of 1.5 in the experiment), because the rate of the \( B^3\Pi \) state decay after the discharge is underestimated.

Figure 3 shows the emission overshoot factor (peak versus steady-state emission intensity) as a function of the shock Mach number, for the delay time of 100 \( \mu \)s. It can be seen that the model overpredicts the first positive emission jump at the low Mach numbers \( M < 3 \), for the reason discussed above, and somewhat underpredicts the second positive emission jump at the high Mach numbers \( M > 2.5 \). Note that at the high Mach numbers, uncertainty in the temperature dependence of the kinetic rates used may be a significant factor affecting the model predictions. Figure 3 also plots the Rankine-Hugoniot density jump, \( \rho_2/\rho_1 \), as well as \( (\rho_2/\rho_1)^2 \), scaled for the relative populations of \( B^3\Pi \) and \( C^3\Pi \) states, respectively, immediately before the shock arrival (see Fig. 2). It can be seen that the predicted \( B \rightarrow A \) emission overshoot scales approximately as \( \rho_2/\rho_1 \), while the predicted \( C \rightarrow B \) emission jump scales close to \( (\rho_2/\rho_1)^2 \). The qualitative reason for this behavior, pointed out by Naidis, is consistent with the results of the present calculations. Basically, the \( B^3\Pi \) state is populated by a second-order collisional process of Eq. (2) and quenched also by a second order process of Eq. (9), which results in the linear \( B^3\Pi \) density scaling behind the shock, proportional to the flow compression by the shock. On the other hand, the \( C^3\Pi \) state behind the shock is populated by a second-order process [energy pooling of Eq. (1)] and quenched by a first-order process [radiative decay of Eq. (8)], which is the principal reason for the \( C \rightarrow B \) emission jump scaling as density squared. Figure 4 shows the emission overshoot factor (peak versus steady-state emission intensity) versus the delay time, for the shock Mach number of \( M = 3.4 \). Again, one can see that the present model correctly predicts the \( C \rightarrow B \) emission jump versus time while the \( B \rightarrow A \) emission jump is somewhat overestimated.

Figure 5 shows normalized density distributions of the neutral species (ground state nitrogen molecules) and of the electrons across a propagating shock. It can be seen that the electrons “leak” across the shock front due to their rapid diffusion, which creates a plasma “precursor” upstream of the shock.
the shock, as discussed in our previous paper.\(^8\) Although the ions follow the rapidly diffusing electrons across the shock, charge separation and local ambipolar electric field are generated upstream of the shock front, due to the large disparity between the electron and the ion diffusion coefficients. Figure 6 shows the reduced electric field, \(E/N\), in the charge separation layer. At these conditions, the double layer thickness is of the order of several Debye lengths, \(l_D\sim 0.3\ \text{mm}\) \((l_D/L\sim 0.006)\). Although it can be seen that the ambipolar electric field in the double layer is quite strong, up to \(E/N \sim 10^{-15}\ \text{V cm}^2\), the calculations show that it does not result in the electron heating since it points in the direction opposite to the electron flux vector. Instead, the electrons do work against the ambipolar field [see Eq. (A3) in the Appendix], and the electron temperature in the double layer is actually reduced. The results of the present paper are consistent with analytic estimates of Naidis\(^6\) suggesting that emission from the shock can be fully explained by the \(C\)\(^{3}\) II and \(B\)\(^{3}\) II state excitations by the processes of Eq. (1) and Eq. (2), respectively.

Second series of calculations has been conducted for a stationary normal shock in the afterglow of transverse rf discharge sustained in a \(M=1.8\) nitrogen flow, for the conditions of our earlier experiments.\(^2\) In these experiments, significant overshoot of the second positive emission systems was detected behind an oblique shock in front of a wedge model placed in the supersonic test section, both in nitrogen and in \(N_2\)–He mixtures. The emission overshoot visualized the shock which appeared as a pair of “cat’s whiskers,” such as shown in Fig. 7.\(^2\) Note that the width of the supersonic flow in Ref. 2 (in the direction perpendicular to the page) was only about 5 mm, which made resolving the emission intensity pattern straightforward. Basically, in Fig. 7 we are looking at a narrow “slice” of an optically thin flow. In the present paper, we use the kinetic model described in Sec. II to illustrate the emission overshoot mechanism qualitatively. In the calculations, transverse electric field \((E/N=7.3 \times 10^{-16}\ \text{V cm}^2)\) was applied across a 5 mm long region, 5 mm upstream of a normal shock standing in the nitrogen flow. The value of \(E/N\) in the discharge was first estimated from the rms rf voltage, \(E/N \sim 10 \times 10^{-16}\ \text{V cm}^2\), and then adjusted to obtain the average electron density in the discharge close to the value inferred from the rms rf discharge current, \(n_e=3 \times 10^{11}\ \text{cm}^{-3}\)\(^.2\) The flow temperature and pressure were \(P=45\ \text{torr}\) and \(T=200\ \text{K}\), respectively, at the Mach number of \(M=1.8\)\(^2\). At these conditions, vibrational excitation of the ground electronic state does not play a significant effect in the discharge kinetics because flow residence time in the discharge, \(\tau_{\text{res}}\sim 10\ \text{ms}\), is much shorter than the characteristic time for electron impact excitation of vibrational levels, \(\tau_{e\nu}\sim (k_{\text{eto}}n_e)^{-1}\sim [(3 \times 10^{-9}\ \text{cm}^3/\text{s})(3 \times 10^{11}\ \text{cm}^{-3})]^{-1}\sim 1\ \text{ms}\).

Figure 8 shows the steady-state normalized density distributions of \(N_2(X\ ^{1}\Sigma)\), \(N_2(B\ ^{3}\Pi)\), and \(N_2(C\ ^{3}\Pi)\) states across the normal shock. It can be seen that although the ground state density behind the shock remains constant, both \(B\ ^{3}\Pi\) and \(C\ ^{3}\Pi\) states number densities (i.e., the first and second positive system emission intensities in the optically thin case) peak at the location of the shock layer and gradually decay downstream of the shock. In this case, the second positive emission overshoot is close to the density jump across the shock, \(\rho_2/\rho_1\), rather than \((\rho_2/\rho_1)^2\), as occurs at a low pressure (see Fig. 3). At this high number density, \(n=2.2\)
The characteristic times for the $C^3\Pi$ state quenching by radiative decay of Eq. (8), and by a collisional process,

$$N_2(C^3\Pi) + N_2(X^1\Sigma) \rightarrow N_2(B^3\Pi) + N_2(X^1\Sigma),$$

become close, $\sim 0.05 \mu$s. Therefore, at these conditions the $C^3\Pi$ state is both populated and depopulated by second order processes of Eqs. (1) and (10) and therefore scales proportional to the density jump across the shock. The calculations also show that the decay rate of both $B^3\Pi$ and $C^3\Pi$ state populations behind the shock is controlled by the energy pooling process of Eq. (1), which depletes the population of the metastable species, $N_2(A^3\Pi)$. This behavior is qualitatively consistent with the second positive emission distribution behind the oblique shock (see Fig. 7).

Figure 9 shows the electron and the ion density distributions across the shock, as well as the reduced electric field in the double layer formed in the shock front. Note that the ambipolar electric field formed in the double layer at these conditions is almost an order of magnitude lower than in Fig. 6 because of the lower shock Mach number. Also, as discussed above, charge separation and the double layer formation across the shock do not produce additional electron heating and therefore do not affect excited electronic state populations. Again, energy pooling mechanism of Eq. (1) remains the dominant process of population and quenching of $B^3\Pi$ and $C^3\Pi$ states behind the shock. Therefore, the present modeling calculations suggest that this process is responsible for the shock visualization in the supersonic flowing afterglow experiments. Figure 8 also shows that this effect may allow rather precise detection of the shock location in the experiment, since the emission peaks immediately downstream ($\sim 0.01$ mm) of the shock layer.

**IV. SUMMARY**

Kinetic modeling of propagating and stationary normal shocks in nonequilibrium nitrogen afterglow plasma is used to simulate the results of shock emission measurements in nitrogen afterglow. Emission intensity overshoot behind the shock predicted by the model is in satisfactory agreement with the experimental results and is consistent with previous analytic estimates. The model demonstrates that the first and the second positive band emission overshoot behind the shock are produced by energy transfer processes among the triplet electronic states of nitrogen generated in the electric discharge. On the other hand, charge separation and ambipolar electric field produced across the shock layer do not result in electron heating and additional electron impact excitation of electronic states. The calculations show that emission overshoot makes possible accurate detection of a stationary shock layer in supersonic flowing afterglow experiments. Further validation of the present model can be obtained by measuring emission intensity distribution behind a stationary shock using a narrow bandpass filter and a charge coupled device (CCD) camera.

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**APPENDIX: QUASI-ONE-DIMENSIONAL NITROGEN PLASMA FLOW EQUATIONS**

In ground state neutral species,

$$\frac{\partial N}{\partial t} + \frac{\partial (NU)}{\partial x} = 0,$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} = -\frac{1}{MN} \frac{\partial P}{\partial x} + \frac{1}{MN} \frac{\partial}{\partial x} \left[ \frac{4}{3} \frac{\partial U}{\partial x} + \nu_n(u - U) \frac{n_i}{N} + \nu_e(u_e - U) \frac{2m_n e}{M N} \right],$$

$$\frac{3}{2} \left( \frac{\partial T}{\partial t} + U \frac{\partial T}{\partial x} \right) = -T \frac{\partial U}{\partial x} + \frac{1}{N k} \lambda \frac{\partial^2 T}{\partial x^2} + \frac{4}{3} \frac{\mu}{N k} \left( \frac{\partial U}{\partial x} \right)^2 + \frac{3}{2} \left( \delta_{\text{coll}} T_{\text{e}, \text{ne}} - \frac{1}{Pr} T \right),$$

$$P = N k T.$$

In ions,

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = R_i - \frac{n_i}{\tau_{i,a}},$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{1}{M n_i} \frac{\partial P_i}{\partial x} + \frac{1}{M n_i} \frac{\partial}{\partial x} \left[ \frac{4}{3} \frac{\mu_i}{M_i} \frac{\partial u_i}{\partial x} + \frac{e}{M_i} E \right] + \nu_{\text{ion}}(u_i - U),$$

where $T_{\text{e}, \text{ne}}$ is the electron-impact ionization rate of nitrogen.

**FIG. 9.** (Color online) Electron and ion density distributions and reduced electric field distribution across a stationary $M=1.8$ normal shock at the conditions of Fig. 7.
\[
\frac{3}{2} \left( \frac{\partial T_i}{\partial t} + u_i \frac{\partial T_i}{\partial x} \right) = - T_i \frac{\partial u_i}{\partial x} + \frac{1}{n_i k} \frac{\partial}{\partial x} \left[ \lambda_i \frac{\partial T_i}{\partial x} \right] + \frac{4 \mu_i}{3 n_i k} \left( \frac{\partial u_i}{\partial x} \right)^2,
\]

\[P_i = n_i k T_i.\]  

(A2)

In electrons,
\[
\frac{\partial n_e}{\partial t} + \frac{\partial (n_e u_e)}{\partial x} = R_e - \frac{n_e}{\tau_{d,e}},
\]

\[\frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} = - \frac{1}{m n_e} \frac{\partial P_e}{\partial x} + \frac{1}{m n_e} \frac{\partial}{\partial x} \left[ \frac{4 \mu_e}{3} \frac{\partial u_e}{\partial x} \right] - \frac{e E}{m} - \nu_{en}(u_e - U),
\]

\[
\frac{3}{2} \left( \frac{\partial T_e}{\partial t} + u_e \frac{\partial T_e}{\partial x} \right) = - T_e \frac{\partial u_e}{\partial x} + \frac{1}{n_e k} \frac{\partial}{\partial x} \left[ \lambda_e \frac{\partial T_e}{\partial x} \right] + \frac{4 \mu_e}{3 n_e k} \left( \frac{\partial u_e}{\partial x} \right)^2 + \left( \frac{e^2 E_{ex}^2}{m k u_e^2} - \frac{e u_e E}{k} - \frac{3}{2} \delta_{coll} T_e \right) \nu_{en},
\]

\[P_e = n_e k T_e.\]  

(A3)

In Poisson equation,
\[
\frac{d E}{dx} = \frac{e}{\varepsilon_0} (n_i - n_e).
\]  

(A4)

In excited species \((j)\) - \(N_2(A^3\Sigma), N_2(B^1\Pi), N_2(C^3\Pi), N_2(a^1\Sigma)\),
\[
\frac{\partial n_j}{\partial t} + \frac{\partial (n_j U)}{\partial x} = \frac{\partial}{\partial x} \left( D \frac{\partial n_j}{\partial x} \right) + R_j - \frac{n_j}{\tau_j}. \tag{A5}
\]

In nitrogen atoms,
\[
\frac{\partial n}{\partial t} + \frac{\partial (n U)}{\partial x} = \frac{\partial}{\partial x} \left( D \frac{\partial n}{\partial x} \right) + R_a - \frac{n_a}{\tau_a}. \tag{A6}
\]

In Eqs. \((A1)-(A6)\), \(R_e\), \(R_j\), and \(R_a\) are the net rates of changes of electron/ion, excited nitrogen molecule, and nitrogen atom numbers in collisional and radiative processes.