Compressibility effects on turbulence structures of axisymmetric mixing layers

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A real-time flow visualization system that produces 17 images over a time span of 150 μs is used to visualize the mixing layers of Mach 1.3 (Ma = 0.59) and Mach 2.0 (Ma = 0.87) ideally expanded high Reynolds number axisymmetric jets. These image sequences reveal details about the influence of compressibility on the dynamics of turbulence structures. In general, the behavior observed in axisymmetric jets is similar to that observed in planar shear layers at similar convective Mach numbers. In addition, large streamwise vortices are apparent in cross-stream images of the flow. Large-scale structures become more three-dimensional and less organized with increasing compressibility and more difficult to identify and track. Planar space–time correlations are used to track structures as they convect downstream. The histogram of the convective velocity for the Mach 1.3 jet revealed a broad distribution of convective velocities with an ensemble average of 266 m/s, which is much higher than the theoretical prediction of 206 m/s. The Mach 2.0 jet, however, exhibited a bimodal convective velocity distribution with an ensemble average of 402 m/s for the “fast” and 190 m/s for the “slow” mode. These modes are equally spaced from the theoretical convective velocity of 303 m/s. Approximately 2/3 of the measured velocities were in the slow mode. © 2003 American Institute of Physics. [DOI: 10.1063/1.1570829]

I. INTRODUCTION

The understanding of the dynamic behavior of compressible flows has long challenged researchers, as the nature of turbulence structures becomes more complex than in incompressible flows. This change in turbulence characteristics has some very profound effects, the most notable being a major reduction in the shear layer growth rate. In order to categorize this effect, a nondimensional parameter, the convective Mach number, was introduced by Bogdanoff and Papamoschou and Roshko. For two pressure-matched parallel streams with equal specific heat ratios the convective Mach number and convective velocity are given as

\[ M_c = \frac{U_1 - U_2}{a_1 + a_2} = \frac{U_1 - U_{c,i}}{a_1} = \frac{U_{c,i} - U_2}{a_2}, \]

\[ U_{c,i} = \frac{a_1 U_2 + a_2 U_1}{a_1 + a_2}, \]

where \( U_1 \) and \( U_2 \) are the high- and low-speed free stream velocities, \( a_1 \) and \( a_2 \) are the speeds of sound and \( U_{c,i} \) is the theoretical isentropic convective velocity. Recent experiments, however, have shown that for \( M_c > 0.5 \) structures do not convect at the speed indicated in Eq. (1). This deviation has been described by the stream selection rule which states that supersonic–subsonic mixing layers will exhibit a convective velocity that is higher than the isentropic prediction. Conversely, for supersonic–supersonic mixing layers, the convective velocity will be lower than the isentropic value. Nonetheless, the convective Mach number as defined in Eq. (1) is still commonly used as a parameter to represent the degree of compressibility.

The role of compressibility in the formation and development of turbulence structures has been experimentally studied using flow visualization in planar shear layers, base flows, and jets. For \( M_c < 0.5 \), turbulence structures are similar to the incompressible structures observed by Brown and Roshko. They are two-dimensional, coherent, and appear as large spanwise rollers. Events such as pairing are commonly seen and easy to follow. The velocity of such large-scale structures is relatively accurately predicted by Eq. (2).

For \( M_c > 1.0 \), the flow is highly compressible and turbulence structures exhibit very little resemblance to their incompressible counterparts. Structures are smaller in scale, difficult to identify, more three-dimensional and lack coherence. Convective velocity measurements also deviate greatly from Eq. (2) and take the form of either a fast or a slow mode. At very high convective Mach numbers (\( M_c > 2 \)), the formation of a co-layer has been experimentally observed with associated convective velocities. At these higher convective Mach numbers, linear stability analyses and numerical simulations predict the emergence of additional instability modes besides the well known Kelvin–Helmholtz instability. These studies also indicate the dominance of three-dimensional modes over two-dimensional.

The compressibility range of \( 0.5 < M_c < 1.0 \), however, marks a transition between these two distinct regimes and is encountered in a large variety of practical flow fields. The flow becomes more three-dimensional as \( M_c \) is increased.
from 0.5 toward 1.0, but elements of both incompressible and compressible flows are present, particularly at the lower end of the range. The convective velocity of structures also deviates from Eq. (2). The goal of this work is to shed additional light on the dynamics of turbulence structures in this regime of compressibility. In particular, this work presents data obtained from Mach 1.3 \((M_c = 0.59)\) and 2.0 \((M_c = 0.87)\) axisymmetric jets using a real-time flow visualization system that can acquire 17 instantaneous images of the flow over a 150 \(\mu s\) time span. The data are analyzed both for qualitative and quantitative changes in the nature of turbulence structures. A planar space–time correlation is used to track structures as they convect downstream in the mixing layer, determining their coherence and the changes in convective velocity with increasing compressibility.

### II. FACILITY AND DIAGNOSTICS

All experiments were conducted at The Ohio State University’s Gas Dynamics and Turbulence Laboratory (GDTL). The facility consists of a jet stand and stagnation chamber to which a variety of nozzles may be attached. Air is supplied from two four-stage compressors; it is filtered, dried and stored in two cylindrical tanks with a total capacity of 42.5 \(m^3\) at 16.5 MPa (1600 ft\(^3\) at 2500 psi). The stagnation chamber contains a perforated plate and two screens of varying porosity to condition the flow to be as uniform as possible prior to entering the nozzle. The facility has been described in more detail elsewhere.\(^{24,25}\) The two axisymmetric nozzles have design Mach numbers of 1.3 and 2.0, and exit diameters of 25.4 mm. The diverging portions of the nozzles were designed using the method of characteristics for uniform flow at the exit. The Mach 2.0 nozzle had a lip thickness of <1 mm and the Mach 1.3 nozzle a slightly thicker lip of 2.5 mm. The jets exhaust into an anechoic chamber and exit the facility through a large bell-mouth at the opposite end of the chamber. Pressure in the stagnation chamber is held constant through each set of experiments.

The Mach numbers of the two nozzles were experimentally determined using a pitot probe to be 1.28 and 2.06, respectively, but will subsequently be referred to as Mach 1.3 and 2.0. The associated Reynolds numbers based on nozzle diameter are \(1.0 \times 10^6\) and \(2.6 \times 10^6\), respectively. The jets’ boundary layers were not measured directly, but are estimated to be laminar and have a momentum thickness, \(\theta\), on the order of 0.1 mm. The Reynolds number based on mixing layer thickness at the imaging locations are approximately \(6.8 \times 10^5\) and \(1.8 \times 10^6\) for the Mach 1.3 and 2.0 jets, respectively. These values are high enough to expect fully developed turbulent flow within each imaging region.

A pulse burst laser and an ultra high-speed CCD camera were used to take sequences of 17 time-correlated flow visualization images. The pulse burst laser has been described elsewhere\(^{11,26–28}\) and only a short description will be provided here. The laser is a custom-built second generation Nd:YAG laser system that achieves a high repetition rate through the use of a continuous wave (cw) oscillator, dual Pockel’s cell pulse “slicer,” and a series of flashlamp-pumped amplifiers. The output is frequency doubled at 532 nm (green). The laser can create between 1 and 99 pulses over a time span of approximately 150 \(\mu s\) and can operate at a maximum rate of 1 MHz. Due to camera limitations the laser was set to generate 17 pulses with inter-pulse timing, \(\Delta t\), of 10 \(\mu s\) (100 000 fps) for the Mach 1.3 jet and 8 \(\mu s\) (125 000 fps) for the Mach 2.0 jet. Each pulse was 10 ns in duration and contains an average of approximately 5 mJ per pulse.

For flow visualization, the laser beam was formed into a sheet and was directed through various slices in the flow field. Seeding was provided using product formation where water vapor contained in the warm moist ambient air condensed into nanometer-scale droplets upon entrainment into the jet and mixing with the cold and dry air of the jet core. Concerns about the size of the particles formed and the response time of their formation have been previously addressed, and the particles are believed to accurately mark the features of the shear layer.\(^{29}\) Laser light scattered from the condensed water particles was then captured and recorded using a Dalsa 64K1M 12-bit CCD camera. This camera can capture 17 images with a resolution of \(245 \times 245\) pixels at a rate up to 1 MHz. Due to the masking technique used to create the high frame rate, the fill factor of the camera is on the order of 3\% with an effective pixel size of \(-10 \mu m\). Despite this limitation of the camera’s sensitivity, the combination of the laser and camera is still sufficient to visualize the flow.

### III. EXPERIMENTAL CONDITIONS

The laser sheet was arranged to acquire images of both streamwise and cross-stream planes in the flow at a variety of downstream distances ranging from 3 to 12 jet diameters. Table I list important properties of the two axisymmetric jets. Defining the potential core length as the point where the centerline velocity reaches 90\% of the jet exit velocity, data from a previous study conducted by Hileman and Samimy.\(^{24}\)

| TABLE I. Important parameters of the two axisymmetric jets studied. |
|-----------------|-----------------|
| Mach number     | Mach 1.3        |
|                 | Mach 2.0        |
| Measured Mach number | 1.28           | 2.06           |
| \(Re_0\)        | \(1.02 \times 10^6\) | \(2.6 \times 10^6\) |
| \(T_j\)         | 226 K           | 162 K          |
| \(M_c\)         | 0.59            | 0.87           |
| Potential core length | \(8.4 x/D\) | \(14.6 x/D\) |
| Imaged region   | \(4.25–10.63 x/D\) | \(5.25–11.75 x/D\) |
| \(\Delta t\)     | \(8 \mu s\)     | \(10 \mu s\)   |
| \(\delta_{local}\) | \(17.1 mm\)   | \(17.9 mm\)   |
| \(\tau\)        | 0.225           | 0.234          |
| \(\tau_{total}\) | 3.6             | 3.74           |
| \(U_j\) (fast mode) | \(385 m/s\)   | \(525 m/s\)   |
| \(U_j\) (slow mode) | \(266 m/s\)   | \(402 m/s\)   |
| Number of image sequences taken | 274             | 100            |
on the Mach 1.3 nozzle gives a measured potential core length of 8.8 \times D. Hileman and Samimy\textsuperscript{24} used a different definition and reported a potential core length of approximately 6 \times D. The potential core length of the Mach 2.0 nozzle was not measured, but is estimated to be 14.6 \times D based upon an empirical equation determined by Murakami and Papamoschou.\textsuperscript{20} The equation of Ref. 30 predicts a core length of 8.4 \times D for the Mach 1.3 jet, which is very close to the measured value and thus deemed an accurate means of estimation. The instantaneous length of the potential core will vary in time over a couple of jet diameters. The visual length of the potential core is slightly longer as seed particles will not mark the entire extent of the mixing layer. Convective velocities (to be discussed in subsequent sections) were calculated for downstream distances of 5.5 and 8 jet diameters for the Mach 1.3 and 2.0 jets, respectively. These locations are between 55\% and 65\% of the potential core length and allow structures to develop without interference from the other side of the mixing layer. A nondimensional time, $\tau$, is defined as

$$\tau = \frac{U_j \Delta t}{\delta_{\text{local}}},$$

where $U_j$ is the jet exit velocity, $\delta_{\text{local}}$ is the local mixing layer thickness, and $\Delta t$ is the time between two images obtained using the real-time flow visualization technique. The visual $\delta_{\text{local}}$ was determined from ensemble-averaged images of the mixing layer and defined as the width of the mixing layer between the 25\% of maximum intensity points on either side of the mixing layer. The value of $\tau$ between each image is nearly the same for both jets indicating that similar time spans are being examined for both the Mach 1.3 and 2.0 jets.

IV. PLANAR SPACE-TIME CORRELATION

In order to calculate the convective velocity of time-correlated structures contained in the mixing layer, an algorithm was developed based upon the two-dimensional spatial cross-correlation. The basic idea of the algorithm is to choose a structure in the first frame of a seventeen frame image sequence and use standard cross-correlation techniques to find its position as well its level of correlation in each subsequent frame. The algorithm was written in the MATLAB programming environment and is similar to algorithms developed by other researchers (particularly, Refs. 3, 7, and 31).

Images are pre-processed by subtraction of background and low-pass filtering (3\times3 pixel moving average). Low-pass filtering has been shown by Wainner and Seitzman\textsuperscript{15} to generally improve the success of cross-correlations in tracking structures. Each image is then corrected on a shot-to-shot basis for nonuniformity of the laser sheet intensity and scaled to have values between 0 and 1.

After pre-processing, a template is automatically chosen from the first frame in the sequence of 17 images. This template is then correlated with each of the 17 frames. The signal within the rectangle in the first frame of Fig. 1 is an example of a typical template. The window is centered on the mixing layer at $x/D = 5.5$ and has a length of 3 $\delta_{\text{local}}$ (78 pixels in this case). The transverse size of the window is large enough to include the entire extent of the top half of the mixing layer. Any signal from the bottom half of the mixing layer that might be contained within the template is automatically removed to ensure that the correlation routine only tracks features in the selected half of the mixing layer. As the size and location of the template might affect the convective velocity results, the velocity was calculated for window sizes ranging from 0.5 to 5.0 $\delta_{\text{local}}$ and downstream locations from 5.5 to 9.0 \times D. Neither variable had a significant effect on the average convective velocity and only slightly affected the velocity histograms. It should be emphasized that the procedure is entirely automated. No specific definition of a structure is used; rather, a turbulence structure is considered as the pattern within the window. A previous investigation\textsuperscript{32} explored the consequences of this in more detail and found automated methods to produce the same results as manual methods. In addition, the structure within the template was analyzed for size, eccentricity, orientation, and centroid.

Next, the two-dimensional spatial cross-correlation is performed to find the location of the structure in each subsequent frame. The correlation is computed using

$$C(n\Delta x, m\Delta y, k) = \sum_{m=-M/2}^{M/2} \sum_{n=0}^{M/2} I'_{M \times N}(x_o, y_o, 1) \times I'_{M \times N}(x_o + n\Delta x, y_o + m\Delta y, k),$$

where $\Delta x$ and $\Delta y$ denote the streamwise and transverse shift of the template, $k$ indicates the frame being searched, $C$ is the correlation level, $M \times N$ is the template size, $I' = I - \langle I \rangle$, $\langle \rangle$ denotes the ensemble average and $'$ denotes the fluctuation signal. $x_o$ and $y_o$ indicate the initial position of the template within the entire image. $\Delta x = \Delta y$ and is the scale of one pixel, which is 635 and 655 \textmu m for the Mach 1.3 and 2.0 jets, respectively. The value of $C$ is then normalized so that it takes a value of 1.0 for perfect correlation and -1.0 for perfect anticorrelation. The correlation is only calculated for physically realistic shifts in structure position. Thus $n$ is only varied between 0 (0 m/s) and $U_j^2(k - 1) \times t_{\text{sep}}/\Delta x$, which corresponds to the number of pixels the structure would move traveling at $U_j$.

In this fashion, a total of 17 two-dimensional cross-correlation matrices are constructed for each image sequence (one for each image including the first frame correlated with itself). This procedure is then repeated for each image sequence in the overall data set. At this point, the data for each sequence of images can be analyzed independently or ensemble averaged over many image sequences to get average properties of structures. In either case, the convective velocity is determined by finding the location of maximum correlation in each of the 17 frames. This process is rather straightforward for the first few frames where the correlation level is fairly high ($>0.6$), but somewhat problematic for the later images in the sequence for which the correlation levels are lower and other features in the flow may create a higher correlation level at different locations. This problem is circumvented by rejecting all data that lies outside of physi-
cally realistic bounds (i.e., the structure does not move backward or faster than the core flow). If no peak can be identified within these bounds, the routine is stopped and the convective velocity is determined from the locations already identified. This process of identifying peak locations results in vectors of structure location, $x$, and time, $t$, which can be plotted on an $x$-$t$ diagram. The slope of a least squares linear fit through these points gives the mean convective velocity over the sequence. This process of calculating convective velocity is performed on both ensemble averages of the correlation matrices and on individual image sets. For the ensemble averages, an accurate average convective velocity of structures is calculated. For instantaneous realizations, a histogram of convective velocities of structures is calculated. For instantaneous realizations, a histogram of convective velocities is determined.

For the ensemble averages, the linear curve fit through the points is very good with $R^2$ values (also known as coefficient of determination) greater than 0.99. Thus the average convective velocity calculations are felt to be quite accurate. Instantaneous realizations of the convective velocity, however, are more difficult to determine due to decreasing correlation levels over time, low camera resolution (structures typically only move 2–5 pixels between frames) and possibly a change in convective velocity with time. In order to ensure accurate instantaneous convective velocity measurements, an algorithm was developed to identify and reject errant points. The basic idea of the algorithm is that all points on the $x$-$t$ diagram must be within a $\pm 0.1 U_j$ boundary of the calculated convective velocity. Any points lying outside of this boundary (and all points later in time) are rejected. This restriction is somewhat arbitrary, but should allow for some acceleration–deceleration of structures while ensuring that the correct structure is being tracked. With this restriction, the number of points that makes up any given measurement can vary, but provides a good linear fit. Thus, we feel that our measurements are accurate within 39 and 53 m/s for the Mach 1.3 and 2.0 jets, respectively. For both jets, approximately 25% of the measurements were rejected due to these criteria. For less stringent restrictions, more data points were obtained with the general characteristics of the histograms remaining the same. Conversely, for more stringent restrictions, fewer data points were obtained, but the general characteristics of the histograms were also the same. Thus, the conclusions reached based on these measurements will be the same regardless of the criteria chosen. These criteria were not applied or needed in the analysis of the ensemble-averaged data.

V. RESULTS

A. Flow visualization

Figure 1 is a sequence of seven images (out of a full sequence of 17) taken of the Mach 1.3 ($M_c = 0.59$) jet. Flow is from left to right and the bright regions correspond to areas where moisture in the entrained ambient air into the mixing layer has condensed. Thus, only the mixing layer is being visualized. The analysis of the seven images in Fig. 1 is greatly aided by the addition of 10 other frames that can be played in a movie format. The movie format makes the de-

FIG. 1. A typical streamwise image sequence of the Mach 1.3 ($M_c = 0.59$) axisymmetric jet.

FIG. 2. A typical cross-stream image sequence of the Mach 1.3 ($M_c = 0.59$) axisymmetric jet at $x/D = 6$. 
 development and interaction of structures within the mixing layer much easier to visualize. As seen in this image (particularly in the upper half of the mixing layer in the sixth image) and previously reported results, one observes only the occasional appearance of structures that resemble the familiar core and braid regions associated with structures in incompressible shear layers. These structures do not appear to be globally organized with respect to other structures in the shear layer as they do in incompressible shear layers. Furthermore, the superposition of many smaller scales is evident by the jagged edges at the high- and low-speed boundaries of the mixing layer. Cross-stream image sequences reveal that the turbulence structures are quite three-dimensional. This is evident in Fig. 2 taken at \( x/D = 6.0 \), where the mixing layer is quite undulated and consists of a variety of structures, including streamwise vortices, and scales. The inner edge of the mixing layer evolves quite rapidly (best noticed by examining the shape and position of the jet core) while the outer edge of the mixing layer evolves slowly. Streamwise vortices are also deemed to play a key role in the development of the mixing layer as inferred by the large swirling motions of fluid observed in the cross-stream movies, but difficult to see in Fig. 2 due to the lack of 13 images from the full sequence. Direct evidence of this motion is seen in the mushroom vortex pair in the upper left corner of the first two images, and indirect evidence is seen from fingering of the mixing layer in several locations in the images of Fig. 2. These overall observations agree quite well with the observations made in studies of planar shear layers, except for the more prominent presence of streamwise vortices in the present case.

The technique used here provides some additional insights concerning the evolution of and interaction between structures that have not previously been available. Figure 1 depicts an event that typifies the dynamics of structures in this compressible mixing layer. In the first frame, three structures are labeled, “A,” “B,” and “C.” These structures do not appear to be the same as structures seen in incompressible flows but distinguish themselves from the rest of the mixing layer and are separated by small braid-like regions. In the third frame (which is really the fifth in the full sequence of images), structures “A” and “B” are slightly tilted and stretched in the direction of the shear. Structure “C,” meanwhile, appears roughly the same as it did in frame 1, possibly tilted and stretched slightly. In the fifth frame (ninth in the full sequence) structure “B” is dramatically tilted and stretched from its original shape and now overlaps “A” on the low-speed side of the mixing layer and “C” on the high-speed side. In the seventh frame (13th in the full sequence), no evidence of structure “B” exists as it has been torn apart by the pairing interaction with “A” and “C.” Two identifiable structures remain that are labeled “A+B” and “B+C” to indicate their origin. The developments characterized in Fig. 1 can be generalized and decomposed into the basic processes of tilt, stretch, tear and pair. After viewing hundreds of movies, it is quite clear that the processes demonstrated in Fig. 1 are very common and take place throughout the mixing layer on a wide variety of scales. Due to the inherently rapid nature of these events, these mechanisms have only been speculated upon and not shown in such detail until this point. These results could help in clarifying the mechanisms responsible for the growth and destruction of turbulence structures.

Figure 3 shows four images (out of a sequence of 17) that are typical for the Mach 2.0 \((M_c = 0.87)\) jet. Some of the features observed in the \( M_c = 0.59 \) case, as well as some major differences, are observed. For example, a structure is marked “B” in the first frame, which appears to be connected to the structure just upstream of it by a thin braid-like region of fluid. This structure, however, is not as distinct as structures seen in Fig. 1 and evolves much more rapidly as it is difficult to identify beyond the second frame (the fifth in the complete sequence). The tilting, stretching, tearing and
pairing events depicted in Fig. 1 also occur frequently, but are more difficult to follow in still images as the structures undergoing the processes are not as easy to identify and the events appear to happen more rapidly. As in the Mach 1.3 case, cross-stream image sequences indicate the presence of streamwise vortices through the large swirling motions that are observed in the mixing layer. The cross-stream images also reveal the three-dimensional nature of structures. An example of this is in Fig. 4, which presents four images (out of 17) for the Mach 2.0 jet at \( x/D = 9.0 \). The mixing layer is quite wavy. A large structure is apparent in the top half of the first image, but disappears in the remaining images. Unlike the Mach 1.3 images, however, the shape and position of the core does not fluctuate as much and is more stable. This complements the observation that large-scale structures become less coherent and distinct with respect to the rest of the mixing layer. Again, the presence of streamwise vorticity can be inferred from the mushroom shape structures seen in the lower left of images in Fig. 4.

The streamwise image sequences also reveal details concerning the three-dimensional nature of structures. Namely, many of the image sequences show the appearance of fluid from out of the page. This is demonstrated by the fluid structure marked “A” in Fig. 3. In the first image, this fluid appears as a small patch of fluid detached from the rest of the mixing layer. At later times, this fluid element appears to grow in size and still is distinguishable. It is interesting to note that a manual measurement of this feature’s velocity is approximately 440 m/s. It cannot be determined from these images whether the growth of structure “A” is an actual growth of a structure or if it results from the passing of a structure with an azimuthal as well as a longitudinal motion through the image plane. This type of motion, however, is much more common in the Mach 2.0 jet than it is in the Mach 1.3 jet. As in the \( M_c = 0.59 \) case, these observations agree well with observations made on planar shear layers.\(^2,4–7,12–15\)

**B. Space–time correlation results**

We will first examine the ensemble average cross correlation results. Figure 5 shows the correlation levels for the Mach 1.3 data (ensemble average of 249 image sequences) at various time separations. Figure 5 is similar to a conventional space–time correlation graph, except for the change in the role of space and time—in the conventional case, data are obtained at discrete streamwise separations. Each curve is a streamwise slice through the maximum point in the two-dimensional spatial cross-correlation data for the given time separation. By definition, the maximum correlation level for the \( \Delta t = 0 \) case is located at a zero streamwise shift, with subsequent decrease with increasing streamwise distance. For a time separation of 10 \( \mu s \), the maximum correlation level has dropped to about 0.83 and is located just under 3 mm downstream. For increasing time separations, the correlation level continues to drop and the peak broadens. At \( \Delta t = 140 \mu s \), the maximum correlation is about 0.18 and the peak is quite broad. The locations of these peaks are plotted on an \( x-t \) diagram in Fig. 6. The slope of the linear fit through these points is the average convective velocity for the Mach 1.3 jet. The curve fit is very good and the average convective velocity is 266 m/s. This velocity is well above the theoretical prediction [see Eq. (2)] of 206 m/s.

Figure 7 is a plot of the space–time correlations (ensemble average of 100 image sequences) for the Mach 2.0 jet at various time separations. A trend similar to that of the Mach 1.3 case is initially observed, as the maximum correlation decreases and the peak becomes increasingly broad for increased time separations. Starting with a time separation of 80 \( \mu s \), however, a very interesting event happens. This broad peak can now be clearly seen as consisting of two peaks. By individually charting the location of both peaks, two convective velocities can be calculated, a fast mode and a slow mode. This is shown in the \( x-t \) diagram of Fig. 8. The points marking the fast mode do not appear until a time separation of 72 \( \mu s \). The linear fit to these points is quite good and the average convective velocity for the fast mode is 402 m/s while that of the slow mode is 190 m/s. The theoretical value for the convective velocity is calculated as 303 m/s, which is about halfway between the fast mode and the slow mode. As mentioned earlier, both the fast and slow modes appeared no
matter what template size or location was used. The average velocity also did not change by more than 20 m/s. The convective velocity does decrease slightly as the template is located further downstream, as one would expect, due to the drop in the jet centerline velocity.

Figure 9 is a plot of the maximum correlation levels vs nondimensional time for both jets. The nondimensional time was defined in Eq. \((2)\). The normalization collapses the results onto a single curve very well, except for the fast mode, which has a slightly lower correlation level.

Additional details can be obtained about the convective velocity by examining the instantaneous space–time correlation data. Figure 10 is a histogram of instantaneous convective velocities observed for the Mach 1.3 jet. These data include measurements from both the upper and lower halves of the mixing layer and consist of 354 cases. Approximately 29% of the 498 possible measurements were rejected due to the criteria described earlier although histograms with more realizations reveal the same general features. The peak of the histogram is centered at 250 m/s, which is close to the ensemble average convective velocity of 266 m/s, given earlier. As mentioned earlier, the estimated accuracy of each measurement is approximately 39 m/s. The shape of the histogram is not Gaussian and is skewed toward lower velocities.

VI. DISCUSSION

The flow visualization results presented here generally agree with other reported flow visualization studies of compressibility effects in shear layers.\(^7,8,12-14\) The general observation is that with increasing convective Mach number, structures become less organized and more three-dimensional. In addition to these observations, double-pulsed flow visualizations\(^7,8,13\) indicate a suppression of interaction between turbulence structures. This observation is generally supported by this work where events such as pairing were less frequent and more difficult to follow at \(M_c = 0.87\) than they were at 0.59. It is important to note, however, that a complete suppression of pairing events was not observed, in contrast to observations made using double-pulsed imaging. The use of double-pulse flow visualizations, however, is limited in its ability to observe dynamic flow features. The observations made earlier in Figs. 1 and 3 should provide more clarification of the dynamics of the mixing layer with increasing compressibility. The observation of streamwise vortices in cross-stream images of these flow fields is also quite significant and may explain some differences between axisymmetric and planar shear layers.

Another observation in the double-pulsed imaging studies was the decrease in coherence of structures with changing compressibility.\(^7,8,13\) In general, the lifetime of structures was seen to decrease with increasing compressibility. This observation was based on an individual’s ability to follow a structure within a double-pulse sequence (often times with rather
large time separations). The ability to capture 17 frames of structure evolution, however, gives us the unique ability to quantitatively examine the lifetime of a structure. An average structure’s lifetime is best represented by Fig. 9, which is a plot of correlation level vs nondimensional time. The close match between the curves for \( M_c = 0.59 \) and 0.87 seems to indicate that structures in either flow field evolve at approximately the same rate, when measured in units of nondimensional time. This apparent contradiction with other studies may be due to a general change in a structure’s appearance that causes it to be less distinct relative to other features in the flow with increasing compressibility. The use of a mathematical tool like space–time correlation eliminates any subjectivity involved. Furthermore, different flow visualization techniques (PLIF, product formation, etc.) may highlight different features of the flow that will result in different conclusions. This technique needs to be applied to more flow fields, however, before any general conclusions can be made.

The convective velocity results presented here are unique and worthy of some additional discussion. The deviation of the convective velocity from the theoretical isentropic value was expected and has been observed by many other researchers.\(^3\)\(^4\)\(^5\)\(^6\)\(^7\)\(^8\) For the Mach 1.3 jet, we observe a fast mode in agreement with the stream selection rule. For the Mach 2.0 jet, a fast mode is also observed. Contrary to the stream selection rule, however, a slow mode was also observed for the same flow field. This is a quite surprising result that deserves some more discussion.

As discussed briefly in the Introduction, the appearance of “outer” modes at higher compressibility levels has been observed both analytically and experimentally, but at much higher \( M_c \) than examined in this study. Tam and Hu\(^9\) used a vortex sheet model to show that an axisymmetric flow could support three families of instability waves for \( M_c > 1.0 \). These three families consist of the familiar Kelvin–Helmholtz instability wave that dominates subsonic flows and two additional waves that have supersonic and subsonic phase velocities. Jackson and Grosch\(^20\) and Grosch and Jackson\(^21\) performed two- and three-dimensional stability analyses of compressible shear layers. Their results showed that for low compressibility conditions, the flow is dominated by a single instability mode. At higher compressibility levels, however, there are two groups of unstable waves. These waves have supersonic and subsonic phase speeds. More recently, Day et al.\(^23\) explored the issue of fast and slow modes in more detail using linear stability analysis. They also found the presence of fast and slow modes in addition to the central mode that also exists in incompressible shear layers. All three lines of work predict the presence of faster and slower modes at high compressibility levels. There is also experimental evidence of two different modes at high convective Mach numbers. Rossman et al.\(^10\) measured a fast and slow mode in a \( M_c = 2.33 \) shear layer by measuring the angle of Mach waves emitted on opposite sides of the shear layer. In all of these works, these modes do not become dominant or significant until much higher convective Mach numbers than the 0.87 observed here.

It is unclear if the two modes observed in this study are related to the fast and slow modes observed in other works. A question that needs to be answered is why other convective velocity measurements at similar convective Mach numbers have not yielded similar results. Other experimental studies have reported the existence of either a fast or a slow mode, which has resulted in the stream selection rule. These
works, however, have not observed both fast and slow modes in the same flow field.

To address this issue we took a closer look at the convective velocity measurements made in compressible shear layers with similar convective Mach numbers. The vast majority of these measurements have been conducted on planar shear layers or base flows, both of which are enclosed flows. It is conceivable, albeit speculative, that the influence of a solid boundary (i.e., a wall) around the developing shear layer could create a boundary condition that gives preference to one instability mode over the other. This preferred instability might then dominate the flow and thus mask or eliminate the presence of the other mode. Thus, any measurements would only yield a single convective velocity associated with the preferred mode. The influence of solid boundaries on supersonic flows has been analyzed numerically by Tam and Hu\textsuperscript{33} and Ragab and Sheen,\textsuperscript{34} both of which show that a solid boundary can influence the flow’s development.

Another possibility is that the axisymmetric geometry of the jet might allow for the presence of both modes. This was shown to be the case in Tam and Hu.\textsuperscript{19} Experimental work
on the convective velocity within axisymmetric jets, however, is scarce and the authors are aware of only two other studies. Fourgette et al.\textsuperscript{3} used the same seeding technique used here and two single-pulse Nd:YAG lasers to measure the convective velocity of a Mach 1.5 ($M_c=0.7$) air jet. Only 36 measurements were made and they had an average of 350 m/s, which is greater than the theoretical velocity of 230 m/s for that flow. Murakami and Papamoschou\textsuperscript{7} utilized a two-laser set-up and PLIF imaging of acetone seeded into a Mach 1.5 jet with co-flow to measure the convective velocity. In this system, the lasers formed adjacent laser sheets that were fired with a specified time delay. Images were subsequently recorded on a single detector. Using both air and helium mixtures, they were able to simulate a variety of convective Mach numbers between 0.43 and 0.72. For each case, only a limited number of measurements were made. In all cases a fast mode was detected with the exception of the lower compressibility results, which matched the theoretical value quite well. Both of these studies are based on limited datasets and were for convective Mach numbers at or below 0.72. Compared to the convective Mach number of 0.87 in which we observed these modes, the convective Mach numbers examined previously may have not been high enough for simultaneous modes to occur.

Lastly, a distinction must be made between experiments using techniques similar to this study where seed particles were provided via product formation vs studies that used other seeding techniques such as acetone fluorescence in PLIF experiments or condensation in light scattering experiments. The key difference is that product formation seeds areas of the flow where there has been significant mixing, and it highlights the entire structure in the illuminated plane. In contrast, the other techniques mark either the entire high-speed or low-speed stream of the flow, and highlight only a part of the structure in the illuminated plane. Therefore, the former technique, which is used in the present study, is better suited for tracking the evolution of complex structures.

VII. CONCLUSIONS

Real-time flow visualization of the mixing layer of Mach 1.3 ($M_c=0.59$) and Mach 2.0 ($M_c=0.87$) jets revealed details concerning the dynamic characteristics of turbulence structures with increasing compressibility. These flow visualizations confirmed some of the trends found in single and double-pulse flow visualization studies by other researchers. The real-time flow visualization, however, provided additional details concerning these developments. In the case of the Mach 1.3 jet, tearing and pairing occurred frequently. An example was provided where three structures in the mixing layer were seen to merge into two structures as the middle structure was torn, divided and merged with the two structures immediately upstream and downstream. These types of events were also observed in the Mach 2.0 jet, but were less frequent and much more difficult to follow. This was due to increasing three-dimensionality of structures as well as the fact that the structures are less distinct. In both flow fields, the presence of streamwise vortices was observed in cross-stream images of the flow. These vortices seem to play a significant role in mixing layer dynamics, particularly in comparison with planar shear layers.

Two-dimensional space–time correlations were used to reveal more quantitative details of the effects of increased compressibility. The lifetime of structures in both mixing layers was found to be about the same when using a non-dimensional time scale based on the jet’s exit velocity and the local shear layer thickness. Convective velocity measurements were also made using space–time correlations. Ensemble-average results show a single fast mode for the Mach 1.3 jet of 266 m/s, which is significantly higher than the theoretical velocity of 206 m/s. The same technique applied to the Mach 2.0 jet, however, revealed the presence of both a fast and a slow mode. These were measured to be 402 and 190 m/s, respectively. Histograms of instantaneous convective velocity measurements yielded similar features with a broad distribution of convective velocity. Some possible explanations for this dual mode were offered as numerical analyses at higher convective Mach numbers predict the emergence of fast and slow modes. Experiments conducted by other researchers also measure fast and slow modes, but never in the same flow field. Some possible explanations were offered, but this issue is clearly in need of additional study.

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\textsuperscript{12}G. S. Elliott, M. Samimy, and S. A. Armette, “Study of compressible